Mathematica 11.3 Integration Test Results

Test results for the 156 problems in "1.2.3.4 (f x)^m (d+e x^n)^q (a+b x^n+c x^(2 n))^p.m"

Problem 12: Result is not expressed in closed-form.

$$\int \frac{d+e x^3}{x \left(a+b x^3+c x^6\right)} \, dx$$

Optimal (type 3, 78 leaves, 7 steps):

$$\frac{(b d-2 a e) \text{ArcTanh}\left[\frac{b+2 c x^3}{\sqrt{b^2-4 a c}}\right]}{3 a \sqrt{b^2-4 a c}}+\frac{d \text{Log}[x]}{a}-\frac{d \text{Log}\left[a+b x^3+c x^6\right]}{6 a}$$

Result (type 7, 80 leaves):

$$\frac{d \text{Log}[x]}{a}-\frac{\text{RootSum}\left[a+b \#1^3+c \#1^6\&,\frac{b d \text{Log}[x-\#1]-a e \text{Log}[x-\#1]+c d \text{Log}[x-\#1] \#1^3}{b+2 c \#1^3}\&\right]}{3 a}$$

Problem 13: Result is not expressed in closed-form.

$$\int \frac{d+e x^3}{x^4 \left(a+b x^3+c x^6\right)} \, dx$$

Optimal (type 3, 112 leaves, 7 steps):

$$-\frac{d}{3 a x^3}-\frac{\left(b^2 d-2 a c d-a b e\right) \text{ArcTanh}\left[\frac{b+2 c x^3}{\sqrt{b^2-4 a c}}\right]}{3 a^2 \sqrt{b^2-4 a c}}-\frac{(b d-a e) \text{Log}[x]}{a^2}+\frac{(b d-a e) \text{Log}\left[a+b x^3+c x^6\right]}{6 a^2}$$

Result (type 7, 130 leaves):

$$-\frac{d}{3 a x^3}+\frac{(-b d+a e) \text{Log}[x]}{a^2}+\frac{1}{3 a^2}\text{RootSum}\Big[a+b \#1^3+c \#1^6\&,\frac{1}{b+2 c \#1^3}\Big(b^2 d \text{Log}[x-\#1]-a c d \text{Log}[x-\#1]-a b e \text{Log}[x-\#1]+b c d \text{Log}[x-\#1] \#1^3-a c e \text{Log}[x-\#1] \#1^3\Big)\&\Big]$$

Problem 14: Result is not expressed in closed-form.

$$\int \frac{x^4 \left(d + e x^3\right)}{a + b x^3 + c x^6} \, dx$$

Optimal (type 3, 723 leaves, 14 steps):

$$\frac{e x^2}{2 c} - \frac{\left(c d - b e - \frac{b c d - b^2 e + 2 a c e}{\sqrt{b^2 - 4 a c}}\right) \text{ArcTan}\left[\frac{1 - \frac{2\, 2^{1/3} c^{1/3} x}{\left(b - \sqrt{b^2 - 4 a c}\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}\, c^{5/3} \left(b - \sqrt{b^2 - 4 a c}\right)^{1/3}} -$$

$$\frac{\left(c d - b e + \frac{b c d - b^2 e + 2 a c e}{\sqrt{b^2 - 4 a c}}\right) \text{ArcTan}\left[\frac{1 - \frac{2\, 2^{1/3} c^{1/3} x}{\left(b + \sqrt{b^2 - 4 a c}\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}\, c^{5/3} \left(b + \sqrt{b^2 - 4 a c}\right)^{1/3}} -$$

$$\frac{\left(c d - b e - \frac{b c d - b^2 e + 2 a c e}{\sqrt{b^2 - 4 a c}}\right) \text{Log}\left[\left(b - \sqrt{b^2 - 4 a c}\right)^{1/3} + 2^{1/3} c^{1/3} x\right]}{3 \times 2^{2/3} c^{5/3} \left(b - \sqrt{b^2 - 4 a c}\right)^{1/3}} -$$

$$\frac{\left(c d - b e + \frac{b c d - b^2 e + 2 a c e}{\sqrt{b^2 - 4 a c}}\right) \text{Log}\left[\left(b + \sqrt{b^2 - 4 a c}\right)^{1/3} + 2^{1/3} c^{1/3} x\right]}{3 \times 2^{2/3} c^{5/3} \left(b + \sqrt{b^2 - 4 a c}\right)^{1/3}} + \left(\left(c d - b e - \frac{b c d - b^2 e + 2 a c e}{\sqrt{b^2 - 4 a c}}\right)\right.$$

$$\left.\text{Log}\left[\left(b - \sqrt{b^2 - 4 a c}\right)^{2/3} - 2^{1/3} c^{1/3} \left(b - \sqrt{b^2 - 4 a c}\right)^{1/3} x + 2^{2/3} c^{2/3} x^2\right]\right) \Big/$$

$$\left(6 \times 2^{2/3} c^{5/3} \left(b - \sqrt{b^2 - 4 a c}\right)^{1/3}\right) + \left(\left(c d - b e + \frac{b c d - b^2 e + 2 a c e}{\sqrt{b^2 - 4 a c}}\right)\right.$$

$$\left.\text{Log}\left[\left(b + \sqrt{b^2 - 4 a c}\right)^{2/3} - 2^{1/3} c^{1/3} \left(b + \sqrt{b^2 - 4 a c}\right)^{1/3} x + 2^{2/3} c^{2/3} x^2\right]\right) \Big/$$

$$\left(6 \times 2^{2/3} c^{5/3} \left(b + \sqrt{b^2 - 4 a c}\right)^{1/3}\right)$$

Result (type 7, 88 leaves):

$$\frac{1}{6 c}$$

$$\left(3 e x^2 - 2 \,\text{RootSum}\left[a + b \,\#1^3 + c \,\#1^6 \,\&, \frac{a e \,\text{Log}[x - \#1] - c d \,\text{Log}[x - \#1] \,\#1^3 + b e \,\text{Log}[x - \#1] \,\#1^3}{b \,\#1 + 2 c \,\#1^4} \,\&\right]\right)$$

Problem 15: Result is not expressed in closed-form.

$$\int \frac{x^3 \left(d + e x^3\right)}{a + b x^3 + c x^6} \, dx$$

Optimal (type 3, 718 leaves, 14 steps):

$$\frac{e\,x}{c}-\frac{\left(c\,d-b\,e-\frac{b\,c\,d-b^2\,e+2\,a\,c\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,c^{1/3}\,x}{\left(b-\sqrt{b^2-4\,a\,c}\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\,\sqrt{3}\,c^{4/3}\,\left(b-\sqrt{b^2-4\,a\,c}\right)^{2/3}}-$$

$$\frac{\left(c\,d-b\,e+\frac{b\,c\,d-b^2\,e+2\,a\,c\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,c^{1/3}\,x}{\left(b+\sqrt{b^2-4\,a\,c}\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\,\sqrt{3}\,c^{4/3}\,\left(b+\sqrt{b^2-4\,a\,c}\right)^{2/3}}+$$

$$\frac{\left(c\,d-b\,e-\frac{b\,c\,d-b^2\,e+2\,a\,c\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{Log}\left[\left(b-\sqrt{b^2-4\,a\,c}\right)^{1/3}+2^{1/3}\,c^{1/3}\,x\right]}{3\times 2^{1/3}\,c^{4/3}\,\left(b-\sqrt{b^2-4\,a\,c}\right)^{2/3}}+$$

$$\frac{\left(c\,d-b\,e+\frac{b\,c\,d-b^2\,e+2\,a\,c\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{Log}\left[\left(b+\sqrt{b^2-4\,a\,c}\right)^{1/3}+2^{1/3}\,c^{1/3}\,x\right]}{3\times 2^{1/3}\,c^{4/3}\,\left(b+\sqrt{b^2-4\,a\,c}\right)^{2/3}}-\left(\left(c\,d-b\,e-\frac{b\,c\,d-b^2\,e+2\,a\,c\,e}{\sqrt{b^2-4\,a\,c}}\right)\right.$$

$$\left.\text{Log}\left[\left(b-\sqrt{b^2-4\,a\,c}\right)^{2/3}-2^{1/3}\,c^{1/3}\,\left(b-\sqrt{b^2-4\,a\,c}\right)^{1/3}\,x+2^{2/3}\,c^{2/3}\,x^2\right]\right)\Big/$$

$$\left(6\times 2^{1/3}\,c^{4/3}\,\left(b-\sqrt{b^2-4\,a\,c}\right)^{2/3}\right)-\left(\left(c\,d-b\,e+\frac{b\,c\,d-b^2\,e+2\,a\,c\,e}{\sqrt{b^2-4\,a\,c}}\right)\right.$$

$$\left.\text{Log}\left[\left(b+\sqrt{b^2-4\,a\,c}\right)^{2/3}-2^{1/3}\,c^{1/3}\,\left(b+\sqrt{b^2-4\,a\,c}\right)^{1/3}\,x+2^{2/3}\,c^{2/3}\,x^2\right]\right)\Big/$$

$$\left(6\times 2^{1/3}\,c^{4/3}\,\left(b+\sqrt{b^2-4\,a\,c}\right)^{2/3}\right)$$

Result (type 7, 88 leaves):

$$\frac{e\,x}{c}-\frac{\text{RootSum}\left[a+b\,\#1^3+c\,\#1^6\,\&,\ \frac{a\,e\,\text{Log}[x-\#1]-c\,d\,\text{Log}[x-\#1]\,\#1^3+b\,e\,\text{Log}[x-\#1]\,\#1^3}{b\,\#1^2+2\,c\,\#1^5}\,\&\right]}{3\,c}$$

Problem 16: Result is not expressed in closed-form.

$$\int\frac{x\,\left(d+e\,x^3\right)}{a+b\,x^3+c\,x^6}\,dx$$

Optimal (type 3, 634 leaves, 13 steps):

$$-\frac{\left(e+\frac{2 c d-b e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} c^{1/3} x}{\left(b-\sqrt{b^2-4 a c}\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}\, c^{2/3} \left(b-\sqrt{b^2-4 a c}\right)^{1/3}}-\frac{\left(e-\frac{2 c d-b e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} c^{1/3} x}{\left(b+\sqrt{b^2-4 a c}\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}\, c^{2/3} \left(b+\sqrt{b^2-4 a c}\right)^{1/3}}-$$

$$\frac{\left(e+\frac{2 c d-b e}{\sqrt{b^2-4 a c}}\right) \text{Log}\left[\left(b-\sqrt{b^2-4 a c}\right)^{1/3}+2^{1/3} c^{1/3} x\right]}{3\times 2^{2/3} c^{2/3} \left(b-\sqrt{b^2-4 a c}\right)^{1/3}}-$$

$$\frac{\left(e-\frac{2 c d-b e}{\sqrt{b^2-4 a c}}\right) \text{Log}\left[\left(b+\sqrt{b^2-4 a c}\right)^{1/3}+2^{1/3} c^{1/3} x\right]}{3\times 2^{2/3} c^{2/3} \left(b+\sqrt{b^2-4 a c}\right)^{1/3}}+$$

$$\left(\left(e+\frac{2 c d-b e}{\sqrt{b^2-4 a c}}\right) \text{Log}\left[\left(b-\sqrt{b^2-4 a c}\right)^{2/3}-2^{1/3} c^{1/3} \left(b-\sqrt{b^2-4 a c}\right)^{1/3} x+2^{2/3} c^{2/3} x^2\right]\right) \Big/$$
$$\left(6\times 2^{2/3} c^{2/3} \left(b-\sqrt{b^2-4 a c}\right)^{1/3}\right)+$$
$$\left(\left(e-\frac{2 c d-b e}{\sqrt{b^2-4 a c}}\right) \text{Log}\left[\left(b+\sqrt{b^2-4 a c}\right)^{2/3}-2^{1/3} c^{1/3} \left(b+\sqrt{b^2-4 a c}\right)^{1/3} x+2^{2/3} c^{2/3} x^2\right]\right) \Big/$$
$$\left(6\times 2^{2/3} c^{2/3} \left(b+\sqrt{b^2-4 a c}\right)^{1/3}\right)$$

Result (type 7, 59 leaves):

$$\frac{1}{3} \text{RootSum}\left[a+b\, \#1^3+c\, \#1^6\,\&,\ \frac{d\, \text{Log}[x-\#1]+e\, \text{Log}[x-\#1]\, \#1^3}{b\, \#1+2 c\, \#1^4}\,\&\right]$$

Problem 17: Result is not expressed in closed-form.

$$\int \frac{d+e x^3}{a+b x^3+c x^6}\, dx$$

Optimal (type 3, 634 leaves, 13 steps):

$$-\frac{\left(e+\frac{2 c d-b e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} c^{1/3} x}{\left(b-\sqrt{b^2-4 a c}\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}\, c^{1/3} \left(b-\sqrt{b^2-4 a c}\right)^{2/3}}-\frac{\left(e-\frac{2 c d-b e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} c^{1/3} x}{\left(b+\sqrt{b^2-4 a c}\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}\, c^{1/3} \left(b+\sqrt{b^2-4 a c}\right)^{2/3}}+$$

$$\frac{\left(e+\frac{2 c d-b e}{\sqrt{b^2-4 a c}}\right) \text{Log}\left[\left(b-\sqrt{b^2-4 a c}\right)^{1/3}+2^{1/3} c^{1/3} x\right]}{3\times 2^{1/3} c^{1/3} \left(b-\sqrt{b^2-4 a c}\right)^{2/3}}+$$

$$\frac{\left(e-\frac{2 c d-b e}{\sqrt{b^2-4 a c}}\right) \text{Log}\left[\left(b+\sqrt{b^2-4 a c}\right)^{1/3}+2^{1/3} c^{1/3} x\right]}{3\times 2^{1/3} c^{1/3} \left(b+\sqrt{b^2-4 a c}\right)^{2/3}}-$$

$$\left(\left(e+\frac{2 c d-b e}{\sqrt{b^2-4 a c}}\right) \text{Log}\left[\left(b-\sqrt{b^2-4 a c}\right)^{2/3}-2^{1/3} c^{1/3} \left(b-\sqrt{b^2-4 a c}\right)^{1/3} x+2^{2/3} c^{2/3} x^2\right]\right)\Big/$$
$$\left(6\times 2^{1/3} c^{1/3} \left(b-\sqrt{b^2-4 a c}\right)^{2/3}\right)-$$
$$\left(\left(e-\frac{2 c d-b e}{\sqrt{b^2-4 a c}}\right) \text{Log}\left[\left(b+\sqrt{b^2-4 a c}\right)^{2/3}-2^{1/3} c^{1/3} \left(b+\sqrt{b^2-4 a c}\right)^{1/3} x+2^{2/3} c^{2/3} x^2\right]\right)\Big/$$
$$\left(6\times 2^{1/3} c^{1/3} \left(b+\sqrt{b^2-4 a c}\right)^{2/3}\right)$$

Result (type 7, 61 leaves):

$$\frac{1}{3} \text{RootSum}\left[a+b \#1^3+c \#1^6\,\&,\ \frac{d \,\text{Log}[x-\#1]+e\, \text{Log}[x-\#1]\, \#1^3}{b \#1^2+2 c \#1^5}\,\&\right]$$

Problem 18: Result is not expressed in closed-form.

$$\int \frac{d+e x^3}{x^2 \left(a+b x^3+c x^6\right)} \, dx$$

Optimal (type 3, 653 leaves, 14 steps):

$$-\frac{d}{a\,x}+\frac{c^{1/3}\left(d+\frac{b\,d-2\,a\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,c^{1/3}\,x}{\left(b-\sqrt{b^2-4\,a\,c}\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\,\sqrt{3}\,a\,\left(b-\sqrt{b^2-4\,a\,c}\right)^{1/3}}+\frac{c^{1/3}\left(d-\frac{b\,d-2\,a\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,c^{1/3}\,x}{\left(b+\sqrt{b^2-4\,a\,c}\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\,\sqrt{3}\,a\,\left(b+\sqrt{b^2-4\,a\,c}\right)^{1/3}}+$$

$$\frac{c^{1/3}\left(d+\frac{b\,d-2\,a\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{Log}\left[\left(b-\sqrt{b^2-4\,a\,c}\right)^{1/3}+2^{1/3}\,c^{1/3}\,x\right]}{3\times 2^{2/3}\,a\,\left(b-\sqrt{b^2-4\,a\,c}\right)^{1/3}}+$$

$$\frac{c^{1/3}\left(d-\frac{b\,d-2\,a\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{Log}\left[\left(b+\sqrt{b^2-4\,a\,c}\right)^{1/3}+2^{1/3}\,c^{1/3}\,x\right]}{3\times 2^{2/3}\,a\,\left(b+\sqrt{b^2-4\,a\,c}\right)^{1/3}}-$$

$$\left(c^{1/3}\left(d+\frac{b\,d-2\,a\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{Log}\left[\left(b-\sqrt{b^2-4\,a\,c}\right)^{2/3}-2^{1/3}\,c^{1/3}\left(b-\sqrt{b^2-4\,a\,c}\right)^{1/3}x+2^{2/3}\,c^{2/3}\,x^2\right]\right)\Big/$$
$$\left(6\times 2^{2/3}\,a\,\left(b-\sqrt{b^2-4\,a\,c}\right)^{1/3}\right)-$$
$$\left(c^{1/3}\left(d-\frac{b\,d-2\,a\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{Log}\left[\left(b+\sqrt{b^2-4\,a\,c}\right)^{2/3}-2^{1/3}\,c^{1/3}\left(b+\sqrt{b^2-4\,a\,c}\right)^{1/3}x+2^{2/3}\,c^{2/3}\,x^2\right]\right)\Big/$$
$$\left(6\times 2^{2/3}\,a\,\left(b+\sqrt{b^2-4\,a\,c}\right)^{1/3}\right)$$

Result (type 7, 85 leaves):

$$-\frac{d}{a\,x}-\frac{\text{RootSum}\left[a+b\,\#1^3+c\,\#1^6\,\&,\ \frac{b\,d\,\text{Log}[x-\#1]-a\,e\,\text{Log}[x-\#1]+c\,d\,\text{Log}[x-\#1]\,\#1^3}{b\,\#1+2\,c\,\#1^4}\,\&\right]}{3\,a}$$

Problem 19: Result is not expressed in closed-form.

$$\int\frac{d+e\,x^3}{x^3\,\left(a+b\,x^3+c\,x^6\right)}\,dx$$

Optimal (type 3, 655 leaves, 14 steps):

$$-\frac{d}{2\,a\,x^2}+\frac{c^{2/3}\left(d+\frac{b\,d-2\,a\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,c^{1/3}\,x}{\left(b-\sqrt{b^2-4\,a\,c}\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\,\sqrt{3}\,a\left(b-\sqrt{b^2-4\,a\,c}\right)^{2/3}}+\frac{c^{2/3}\left(d-\frac{b\,d-2\,a\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,c^{1/3}\,x}{\left(b+\sqrt{b^2-4\,a\,c}\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\,\sqrt{3}\,a\left(b+\sqrt{b^2-4\,a\,c}\right)^{2/3}}-$$

$$\frac{c^{2/3}\left(d+\frac{b\,d-2\,a\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{Log}\left[\left(b-\sqrt{b^2-4\,a\,c}\right)^{1/3}+2^{1/3}\,c^{1/3}\,x\right]}{3\times 2^{1/3}\,a\left(b-\sqrt{b^2-4\,a\,c}\right)^{2/3}}-$$

$$\frac{c^{2/3}\left(d-\frac{b\,d-2\,a\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{Log}\left[\left(b+\sqrt{b^2-4\,a\,c}\right)^{1/3}+2^{1/3}\,c^{1/3}\,x\right]}{3\times 2^{1/3}\,a\left(b+\sqrt{b^2-4\,a\,c}\right)^{2/3}}+$$

$$\left(c^{2/3}\left(d+\frac{b\,d-2\,a\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{Log}\left[\left(b-\sqrt{b^2-4\,a\,c}\right)^{2/3}-2^{1/3}\,c^{1/3}\left(b-\sqrt{b^2-4\,a\,c}\right)^{1/3}x+2^{2/3}\,c^{2/3}\,x^2\right]\right)\Big/$$
$$\left(6\times 2^{1/3}\,a\left(b-\sqrt{b^2-4\,a\,c}\right)^{2/3}\right)+$$
$$\left(c^{2/3}\left(d-\frac{b\,d-2\,a\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{Log}\left[\left(b+\sqrt{b^2-4\,a\,c}\right)^{2/3}-2^{1/3}\,c^{1/3}\left(b+\sqrt{b^2-4\,a\,c}\right)^{1/3}x+2^{2/3}\,c^{2/3}\,x^2\right]\right)\Big/$$
$$\left(6\times 2^{1/3}\,a\left(b+\sqrt{b^2-4\,a\,c}\right)^{2/3}\right)$$

Result (type 7, 89 leaves):

$$-\frac{d}{2\,a\,x^2}-\frac{\text{RootSum}\left[a+b\,\#1^3+c\,\#1^6\,\&,\,\frac{b\,d\,\text{Log}[x-\#1]-a\,e\,\text{Log}[x-\#1]+c\,d\,\text{Log}[x-\#1]\,\#1^3}{b\,\#1^2+2\,c\,\#1^5}\,\&\right]}{3\,a}$$

Problem 23: Result is not expressed in closed-form.

$$\int\frac{1-x^3}{x\left(1-x^3+x^6\right)}\,dx$$

Optimal (type 3, 41 leaves, 7 steps):

$$\frac{\text{ArcTan}\left[\frac{1-2\,x^3}{\sqrt{3}}\right]}{3\,\sqrt{3}}+\text{Log}[x]-\frac{1}{6}\,\text{Log}\left[1-x^3+x^6\right]$$

Result (type 7, 44 leaves):

$$\text{Log}[x]-\frac{1}{3}\,\text{RootSum}\left[1-\#1^3+\#1^6\,\&,\,\frac{\text{Log}[x-\#1]\,\#1^3}{-1+2\,\#1^3}\,\&\right]$$

Problem 24: Result is not expressed in closed-form.

$$\int\frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx$$

Optimal (type 3, 31 leaves, 5 steps):

$$-\frac{1}{3 x^3}+\frac{2 \operatorname{ArcTan}\left[\frac{1-2 x^3}{\sqrt{3}}\right]}{3 \sqrt{3}}$$

Result (type 7, 45 leaves):

$$-\frac{1}{3 x^3}-\frac{1}{3} \operatorname{RootSum}\left[1-\#1^3+\#1^6 \&, \frac{\operatorname{Log}[x-\#1]}{-1+2 \#1^3} \&\right]$$

Problem 25: Result is not expressed in closed-form.

$$\int \frac{x^6 \left(1-x^3\right)}{1-x^3+x^6} \, dx$$

Optimal (type 3, 418 leaves, 15 steps):

$$-\frac{x^4}{4}-\frac{\left(i+\sqrt{3}\right) \operatorname{ArcTan}\left[\frac{1+\frac{2 x}{\left(\frac{1}{2}\left(1-i \sqrt{3}\right)\right)^{1/3}}}{\sqrt{3}}\right]}{3\times 2^{1/3} \left(1-i \sqrt{3}\right)^{2/3}}+\frac{\left(i-\sqrt{3}\right) \operatorname{ArcTan}\left[\frac{1+\frac{2 x}{\left(\frac{1}{2}\left(1+i \sqrt{3}\right)\right)^{1/3}}}{\sqrt{3}}\right]}{3\times 2^{1/3} \left(1+i \sqrt{3}\right)^{2/3}}+$$

$$\frac{\left(3+i \sqrt{3}\right) \operatorname{Log}\left[\left(1-i \sqrt{3}\right)^{1/3}-2^{1/3} x\right]}{9\times 2^{1/3} \left(1-i \sqrt{3}\right)^{2/3}}+\frac{\left(3-i \sqrt{3}\right) \operatorname{Log}\left[\left(1+i \sqrt{3}\right)^{1/3}-2^{1/3} x\right]}{9\times 2^{1/3} \left(1+i \sqrt{3}\right)^{2/3}}-$$

$$\frac{\left(3+i \sqrt{3}\right) \operatorname{Log}\left[\left(1-i \sqrt{3}\right)^{2/3}+\left(2 \left(1-i \sqrt{3}\right)\right)^{1/3} x+2^{2/3} x^2\right]}{18\times 2^{1/3} \left(1-i \sqrt{3}\right)^{2/3}}-$$

$$\frac{\left(3-i \sqrt{3}\right) \operatorname{Log}\left[\left(1+i \sqrt{3}\right)^{2/3}+\left(2 \left(1+i \sqrt{3}\right)\right)^{1/3} x+2^{2/3} x^2\right]}{18\times 2^{1/3} \left(1+i \sqrt{3}\right)^{2/3}}$$

Result (type 7, 47 leaves):

$$-\frac{x^4}{4}+\frac{1}{3} \operatorname{RootSum}\left[1-\#1^3+\#1^6 \&, \frac{\operatorname{Log}[x-\#1] \#1}{-1+2 \#1^3} \&\right]$$

Problem 26: Result is not expressed in closed-form.

$$\int \frac{x^4 \left(1-x^3\right)}{1-x^3+x^6} \, dx$$

Optimal (type 3, 382 leaves, 15 steps):

$$-\frac{x^2}{2}+\frac{i\,\mathrm{ArcTan}\left[\frac{1+\frac{2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{1/3}}}{\sqrt{3}}\right]}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{1/3}}-\frac{i\,\mathrm{ArcTan}\left[\frac{1+\frac{2x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{1/3}}}{\sqrt{3}}\right]}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{1/3}}+\frac{i\,\mathrm{Log}\left[\left(1-i\sqrt{3}\right)^{1/3}-2^{1/3}x\right]}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{1/3}}-\frac{i\,\mathrm{Log}\left[\left(1+i\sqrt{3}\right)^{1/3}-2^{1/3}x\right]}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{1/3}}-\frac{i\,\mathrm{Log}\left[\left(1-i\sqrt{3}\right)^{2/3}+\left(2\left(1-i\sqrt{3}\right)\right)^{1/3}x+2^{2/3}x^2\right]}{3\times 2^{2/3}\sqrt{3}\left(1-i\sqrt{3}\right)^{1/3}}+\frac{i\,\mathrm{Log}\left[\left(1+i\sqrt{3}\right)^{2/3}+\left(2\left(1+i\sqrt{3}\right)\right)^{1/3}x+2^{2/3}x^2\right]}{3\times 2^{2/3}\sqrt{3}\left(1+i\sqrt{3}\right)^{1/3}}$$

Result (type 7, 48 leaves):

$$-\frac{x^2}{2}+\frac{1}{3}\,\mathrm{RootSum}\left[1-\#1^3+\#1^6\,\&,\ \frac{\mathrm{Log}[x-\#1]}{-\#1+2\,\#1^4}\,\&\right]$$

Problem 27: Result is not expressed in closed-form.

$$\int \frac{x^3\left(1-x^3\right)}{1-x^3+x^6}\,dx$$

Optimal (type 3, 378 leaves, 14 steps):

$$-x-\frac{i\,\mathrm{ArcTan}\left[\frac{1+\frac{2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{1/3}}}{\sqrt{3}}\right]}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\,\mathrm{ArcTan}\left[\frac{1+\frac{2x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{1/3}}}{\sqrt{3}}\right]}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{i\,\mathrm{Log}\left[\left(1-i\sqrt{3}\right)^{1/3}-2^{1/3}x\right]}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\,\mathrm{Log}\left[\left(1+i\sqrt{3}\right)^{1/3}-2^{1/3}x\right]}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\,\mathrm{Log}\left[\left(1-i\sqrt{3}\right)^{2/3}+\left(2\left(1-i\sqrt{3}\right)\right)^{1/3}x+2^{2/3}x^2\right]}{3\times 2^{1/3}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{i\,\mathrm{Log}\left[\left(1+i\sqrt{3}\right)^{2/3}+\left(2\left(1+i\sqrt{3}\right)\right)^{1/3}x+2^{2/3}x^2\right]}{3\times 2^{1/3}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}$$

Result (type 7, 46 leaves):

$$-x+\frac{1}{3}\,\mathrm{RootSum}\left[1-\#1^3+\#1^6\,\&,\ \frac{\mathrm{Log}[x-\#1]}{-\#1^2+2\,\#1^5}\,\&\right]$$

Problem 28: Result is not expressed in closed-form.

$$\int \frac{x\left(1-x^3\right)}{1-x^3+x^6}\,dx$$

Optimal (type 3, 411 leaves, 13 steps):

$$\frac{\left(i-\sqrt{3}\right)\text{ArcTan}\left[\frac{1+\frac{2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{1/3}}}{\sqrt{3}}\right]}{3\times 2^{2/3}\left(1-i\sqrt{3}\right)^{1/3}}-\frac{\left(i+\sqrt{3}\right)\text{ArcTan}\left[\frac{1+\frac{2x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{1/3}}}{\sqrt{3}}\right]}{3\times 2^{2/3}\left(1+i\sqrt{3}\right)^{1/3}}-$$

$$\frac{\left(3-i\sqrt{3}\right)\text{Log}\left[\left(1-i\sqrt{3}\right)^{1/3}-2^{1/3}x\right]}{9\times 2^{2/3}\left(1-i\sqrt{3}\right)^{1/3}}-\frac{\left(3+i\sqrt{3}\right)\text{Log}\left[\left(1+i\sqrt{3}\right)^{1/3}-2^{1/3}x\right]}{9\times 2^{2/3}\left(1+i\sqrt{3}\right)^{1/3}}+$$

$$\frac{\left(3-i\sqrt{3}\right)\text{Log}\left[\left(1-i\sqrt{3}\right)^{2/3}+\left(2\left(1-i\sqrt{3}\right)\right)^{1/3}x+2^{2/3}x^2\right]}{18\times 2^{2/3}\left(1-i\sqrt{3}\right)^{1/3}}+$$

$$\frac{\left(3+i\sqrt{3}\right)\text{Log}\left[\left(1+i\sqrt{3}\right)^{2/3}+\left(2\left(1+i\sqrt{3}\right)\right)^{1/3}x+2^{2/3}x^2\right]}{18\times 2^{2/3}\left(1+i\sqrt{3}\right)^{1/3}}$$

Result (type 7, 55 leaves):

$$-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\text{Log}[x-\#1]+\text{Log}[x-\#1]\,\#1^3}{-\#1+2\,\#1^4}\&\right]$$

Problem 29: Result is not expressed in closed-form.

$$\int\frac{1-x^3}{1-x^3+x^6}\,dx$$

Optimal (type 3, 411 leaves, 13 steps):

$$-\frac{\left(i-\sqrt{3}\right)\text{ArcTan}\left[\frac{1+\frac{2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{1/3}}}{\sqrt{3}}\right]}{3\times 2^{1/3}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(i+\sqrt{3}\right)\text{ArcTan}\left[\frac{1+\frac{2x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{1/3}}}{\sqrt{3}}\right]}{3\times 2^{1/3}\left(1+i\sqrt{3}\right)^{2/3}}-$$

$$\frac{\left(3-i\sqrt{3}\right)\text{Log}\left[\left(1-i\sqrt{3}\right)^{1/3}-2^{1/3}x\right]}{9\times 2^{1/3}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\text{Log}\left[\left(1+i\sqrt{3}\right)^{1/3}-2^{1/3}x\right]}{9\times 2^{1/3}\left(1+i\sqrt{3}\right)^{2/3}}+$$

$$\frac{\left(3-i\sqrt{3}\right)\text{Log}\left[\left(1-i\sqrt{3}\right)^{2/3}+\left(2\left(1-i\sqrt{3}\right)\right)^{1/3}x+2^{2/3}x^2\right]}{18\times 2^{1/3}\left(1-i\sqrt{3}\right)^{2/3}}+$$

$$\frac{\left(3+i\sqrt{3}\right)\text{Log}\left[\left(1+i\sqrt{3}\right)^{2/3}+\left(2\left(1+i\sqrt{3}\right)\right)^{1/3}x+2^{2/3}x^2\right]}{18\times 2^{1/3}\left(1+i\sqrt{3}\right)^{2/3}}$$

Result (type 7, 57 leaves):

$$-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\text{Log}[x-\#1]+\text{Log}[x-\#1]\,\#1^3}{-\#1^2+2\,\#1^5}\&\right]$$

Problem 30: Result is not expressed in closed-form.

$$\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx$$

Optimal (type 3, 416 leaves, 14 steps):

$$-\frac{1}{x}-\frac{\left(i+\sqrt{3}\right)\text{ArcTan}\left[\frac{1+\frac{2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{1/3}}}{\sqrt{3}}\right]}{3\times 2^{2/3}\left(1-i\sqrt{3}\right)^{1/3}}+\frac{\left(i-\sqrt{3}\right)\text{ArcTan}\left[\frac{1+\frac{2x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{1/3}}}{\sqrt{3}}\right]}{3\times 2^{2/3}\left(1+i\sqrt{3}\right)^{1/3}}-\frac{\left(3+i\sqrt{3}\right)\text{Log}\left[\left(1-i\sqrt{3}\right)^{1/3}-2^{1/3}x\right]}{9\times 2^{2/3}\left(1-i\sqrt{3}\right)^{1/3}}-\frac{\left(3-i\sqrt{3}\right)\text{Log}\left[\left(1+i\sqrt{3}\right)^{1/3}-2^{1/3}x\right]}{9\times 2^{2/3}\left(1+i\sqrt{3}\right)^{1/3}}+\frac{\left(3+i\sqrt{3}\right)\text{Log}\left[\left(1-i\sqrt{3}\right)^{2/3}+\left(2\left(1-i\sqrt{3}\right)\right)^{1/3}x+2^{2/3}x^2\right]}{18\times 2^{2/3}\left(1-i\sqrt{3}\right)^{1/3}}+\frac{\left(3-i\sqrt{3}\right)\text{Log}\left[\left(1+i\sqrt{3}\right)^{2/3}+\left(2\left(1+i\sqrt{3}\right)\right)^{1/3}x+2^{2/3}x^2\right]}{18\times 2^{2/3}\left(1+i\sqrt{3}\right)^{1/3}}$$

Result (type 7, 47 leaves):

$$-\frac{1}{x}-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\text{Log}[x-\#1]\,\#1^2}{-1+2\,\#1^3}\&\right]$$

Problem 31: Result is not expressed in closed-form.

$$\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx$$

Optimal (type 3, 418 leaves, 15 steps):

$$-\frac{1}{2 x^2}+\frac{\left(i+\sqrt{3}\right) \text{ArcTan}\left[\frac{1+\frac{2 x}{\left(\frac{1}{2}\left(1-i \sqrt{3}\right)\right)^{1/3}}}{\sqrt{3}}\right]}{3\times 2^{1/3} \left(1-i \sqrt{3}\right)^{2/3}}-\frac{\left(i-\sqrt{3}\right) \text{ArcTan}\left[\frac{1+\frac{2 x}{\left(\frac{1}{2}\left(1+i \sqrt{3}\right)\right)^{1/3}}}{\sqrt{3}}\right]}{3\times 2^{1/3} \left(1+i \sqrt{3}\right)^{2/3}}-$$
$$\frac{\left(3+i \sqrt{3}\right) \text{Log}\left[\left(1-i \sqrt{3}\right)^{1/3}-2^{1/3} x\right]}{9\times 2^{1/3} \left(1-i \sqrt{3}\right)^{2/3}}-\frac{\left(3-i \sqrt{3}\right) \text{Log}\left[\left(1+i \sqrt{3}\right)^{1/3}-2^{1/3} x\right]}{9\times 2^{1/3} \left(1+i \sqrt{3}\right)^{2/3}}+$$
$$\frac{\left(3+i \sqrt{3}\right) \text{Log}\left[\left(1-i \sqrt{3}\right)^{2/3}+\left(2 \left(1-i \sqrt{3}\right)\right)^{1/3} x+2^{2/3} x^2\right]}{18\times 2^{1/3} \left(1-i \sqrt{3}\right)^{2/3}}+$$
$$\frac{\left(3-i \sqrt{3}\right) \text{Log}\left[\left(1+i \sqrt{3}\right)^{2/3}+\left(2 \left(1+i \sqrt{3}\right)\right)^{1/3} x+2^{2/3} x^2\right]}{18\times 2^{1/3} \left(1+i \sqrt{3}\right)^{2/3}}$$

Result (type 7, 47 leaves):

$$-\frac{1}{2 x^2}-\frac{1}{3} \text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\text{Log}[x-\#1]\,\#1}{-1+2\,\#1^3}\&\right]$$

Problem 33: Result is not expressed in closed-form.

$$\int \frac{1+x^3}{x \left(1-x^3+x^6\right)} \, dx$$

Optimal (type 3, 39 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-2 x^3}{\sqrt{3}}\right]}{\sqrt{3}}+\text{Log}[x]-\frac{1}{6} \text{Log}\left[1-x^3+x^6\right]$$

Result (type 7, 55 leaves):

$$\text{Log}[x]-\frac{1}{3} \text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-2 \,\text{Log}[x-\#1]+\text{Log}[x-\#1]\,\#1^3}{-1+2\,\#1^3}\&\right]$$

Problem 34: Result is not expressed in closed-form.

$$\int \frac{1+x^3}{x-x^4+x^7} \, dx$$

Optimal (type 3, 39 leaves, 8 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-2 x^3}{\sqrt{3}}\right]}{\sqrt{3}}+\text{Log}[x]-\frac{1}{6} \text{Log}\left[1-x^3+x^6\right]$$

Result (type 7, 55 leaves):

$$\text{Log}[x]-\frac{1}{3} \text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-2 \,\text{Log}[x-\#1]+\text{Log}[x-\#1]\,\#1^3}{-1+2\,\#1^3}\&\right]$$

Problem 35: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(d+e x^3\right)^{5/2} \left(a+b x^3+c x^6\right) \, dx$$

Optimal (type 4, 396 leaves, 6 steps):

$$\frac{54 d^2 \left(16 c d^2-58 b d e+667 a e^2\right) x \sqrt{d+e x^3}}{124729 e^2}+\frac{30 d \left(16 c d^2-58 b d e+667 a e^2\right) x \left(d+e x^3\right)^{3/2}}{124729 e^2}+\frac{2 \left(16 c d^2-58 b d e+667 a e^2\right) x \left(d+e x^3\right)^{5/2}}{11339 e^2}-\frac{2 \left(8 c d-29 b e\right) x \left(d+e x^3\right)^{7/2}}{667 e^2}+\frac{2 c x^4 \left(d+e x^3\right)^{7/2}}{29 e}+\left(54\times 3^{3/4} \sqrt{2+\sqrt{3}}\ d^3 \left(16 c d^2-58 b d e+667 a e^2\right) \left(d^{1/3}+e^{1/3} x\right) \sqrt{\frac{d^{2/3}-d^{1/3} e^{1/3} x+e^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) d^{1/3}+e^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) d^{1/3}+e^{1/3} x}{\left(1+\sqrt{3}\right) d^{1/3}+e^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(124729 e^{7/3} \sqrt{\frac{d^{1/3} \left(d^{1/3}+e^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) d^{1/3}+e^{1/3} x\right)^2}} \sqrt{d+e x^3}\right)$$

Result (type 4, 279 leaves):

$$-\frac{1}{124729 (-e)^{7/3} \sqrt{d+e x^3}} 2 \left((-e)^{1/3} \left(d+e x^3\right) \left(d^2 \left(648 c d^2-29 e \left(81 b d+1219 a e\right)\right) x-d e \left(405 c d^2+29 e \left(487 b d+851 a e\right)\right) x^4-11 e^2 \left(781 c d^2+29 e \left(49 b d+23 a e\right)\right) x^7-187 e^3 \left(61 c d+29 b e\right) x^{10}-4301 c e^4 x^{13}\right)-27\, i\, 3^{3/4} d^{10/3} \left(16 c d^2+29 e \left(-2 b d+23 a e\right)\right) \sqrt{(-1)^{5/6} \left(-1+\frac{(-e)^{1/3} x}{d^{1/3}}\right)} \sqrt{1+\frac{(-e)^{1/3} x}{d^{1/3}}+\frac{(-e)^{2/3} x^2}{d^{2/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-e)^{1/3} x}{d^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 36: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(d+e x^3\right)^{3/2} \left(a+b x^3+c x^6\right) \, dx$$

Optimal (type 4, 356 leaves, 5 steps):

$$\frac{18\,d\,\left(16\,c\,d^2-46\,b\,d\,e+391\,a\,e^2\right)\,x\,\sqrt{d+e\,x^3}}{21505\,e^2}+\frac{2\,\left(16\,c\,d^2-46\,b\,d\,e+391\,a\,e^2\right)\,x\,\left(d+e\,x^3\right)^{3/2}}{4301\,e^2}-\frac{2\,\left(8\,c\,d-23\,b\,e\right)\,x\,\left(d+e\,x^3\right)^{5/2}}{391\,e^2}+\frac{2\,c\,x^4\,\left(d+e\,x^3\right)^{5/2}}{23\,e}+\left(18\times 3^{3/4}\,\sqrt{2+\sqrt{3}}\,d^2\,\left(16\,c\,d^2-46\,b\,d\,e+391\,a\,e^2\right)\,\left(d^{1/3}+e^{1/3}\,x\right)\sqrt{\frac{d^{2/3}-d^{1/3}\,e^{1/3}\,x+e^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,d^{1/3}+e^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,d^{1/3}+e^{1/3}\,x}{\left(1+\sqrt{3}\right)\,d^{1/3}+e^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/\left(21505\,e^{7/3}\,\sqrt{\frac{d^{1/3}\,\left(d^{1/3}+e^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,d^{1/3}+e^{1/3}\,x\right)^2}}\,\sqrt{d+e\,x^3}\right)$$

Result (type 4, 249 leaves):

$$-\frac{1}{21505\,(-e)^{7/3}\,\sqrt{d+e\,x^3}}\left(2\,(-e)^{1/3}\,\left(d+e\,x^3\right)\,\left(d\,\left(216\,c\,d^2-23\,e\,\left(27\,b\,d+238\,a\,e\right)\right)\,x-5\,e\,\left(27\,c\,d^2+23\,e\,\left(20\,b\,d+17\,a\,e\right)\right)\,x^4-55\,e^2\,\left(26\,c\,d+23\,b\,e\right)\,x^7-935\,c\,e^3\,x^{10}\right)-9\,\mathrm{i}\,3^{3/4}\,d^{7/3}\,\left(16\,c\,d^2+23\,e\,\left(-2\,b\,d+17\,a\,e\right)\right)\,\sqrt{(-1)^{5/6}\,\left(-1+\frac{(-e)^{1/3}\,x}{d^{1/3}}\right)}\,\sqrt{1+\frac{(-e)^{1/3}\,x}{d^{1/3}}+\frac{(-e)^{2/3}\,x^2}{d^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-e)^{1/3}\,x}{d^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 37: Result unnecessarily involves imaginary or complex numbers.

$$\int\sqrt{d+e\,x^3}\,\left(a+b\,x^3+c\,x^6\right)\,dx$$

Optimal (type 4, 316 leaves, 4 steps):

$$\frac{2\left(16 c d^2-34 b d e+187 a e^2\right) x \sqrt{d+e x^3}}{935 e^2}-\frac{2\left(8 c d-17 b e\right) x\left(d+e x^3\right)^{3/2}}{187 e^2}+$$
$$\frac{2 c x^4\left(d+e x^3\right)^{3/2}}{17 e}+\left(2\times 3^{3/4}\sqrt{2+\sqrt{3}}\ d\left(16 c d^2-34 b d e+187 a e^2\right)\left(d^{1/3}+e^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{d^{2/3}-d^{1/3} e^{1/3} x+e^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) d^{1/3}+e^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) d^{1/3}+e^{1/3} x}{\left(1+\sqrt{3}\right) d^{1/3}+e^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(935 e^{7/3}\sqrt{\frac{d^{1/3}\left(d^{1/3}+e^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) d^{1/3}+e^{1/3} x\right)^2}}\sqrt{d+e x^3}\right)$$

Result (type 4, 219 leaves):

$$-\left(\left(2\left((-e)^{1/3} x\left(d+e x^3\right)\left(-17 e\left(3 b d+11 a e+5 b e x^3\right)+c\left(24 d^2-15 d e x^3-55 e^2 x^6\right)\right)-i\, 3^{3/4} d^{4/3}\right.\right.\right.$$
$$\left(16 c d^2+17 e\left(-2 b d+11 a e\right)\right)\sqrt{(-1)^{5/6}\left(-1+\frac{(-e)^{1/3} x}{d^{1/3}}\right)}\sqrt{1+\frac{(-e)^{1/3} x}{d^{1/3}}+\frac{(-e)^{2/3} x^2}{d^{2/3}}}$$
$$\left.\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-e)^{1/3} x}{d^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(935\,(-e)^{7/3}\sqrt{d+e x^3}\right)\right)$$

Problem 38: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b x^3+c x^6}{\sqrt{d+e x^3}}\,dx$$

Optimal (type 4, 278 leaves, 3 steps):

$$-\frac{2\,(8\,c\,d-11\,b\,e)\,x\,\sqrt{d+e\,x^3}}{55\,e^2}+\frac{2\,c\,x^4\,\sqrt{d+e\,x^3}}{11\,e}+\left(2\,\sqrt{2+\sqrt{3}}\,\left(16\,c\,d^2-22\,b\,d\,e+55\,a\,e^2\right)\,\left(d^{1/3}+e^{1/3}\,x\right)\sqrt{\frac{d^{2/3}-d^{1/3}\,e^{1/3}\,x+e^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,d^{1/3}+e^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,d^{1/3}+e^{1/3}\,x}{\left(1+\sqrt{3}\right)\,d^{1/3}+e^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/\left(55\times 3^{1/4}\,e^{7/3}\,\sqrt{\frac{d^{1/3}\,\left(d^{1/3}+e^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,d^{1/3}+e^{1/3}\,x\right)^2}}\,\sqrt{d+e\,x^3}\right)$$

Result (type 4, 194 leaves):

$$\frac{2\,\sqrt{d+e\,x^3}\,\left(-8\,c\,d\,x+11\,b\,e\,x+5\,c\,e\,x^4\right)}{55\,e^2}+\left(2\,i\,d^{1/3}\,\left(16\,c\,d^2+11\,e\,\left(-2\,b\,d+5\,a\,e\right)\right)\,\sqrt{(-1)^{5/6}\,\left(-1+\frac{(-e)^{1/3}\,x}{d^{1/3}}\right)}\,\sqrt{1+\frac{(-e)^{1/3}\,x}{d^{1/3}}+\frac{(-e)^{2/3}\,x^2}{d^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-e)^{1/3}\,x}{d^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\Bigg/\left(55\times 3^{1/4}\,(-e)^{7/3}\,\sqrt{d+e\,x^3}\right)$$

Problem 39: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,x^3+c\,x^6}{\left(d+e\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 289 leaves, 3 steps):

$$\frac{2\left(c d^2-b d e+a e^2\right) x}{3 d e^2 \sqrt{d+e x^3}}+\frac{2 c x \sqrt{d+e x^3}}{5 e^2}-\left(2 \sqrt{2+\sqrt{3}}\left(16 c d^2-5 e(2 b d+a e)\right)\left(d^{1/3}+e^{1/3} x\right) \sqrt{\frac{d^{2/3}-d^{1/3} e^{1/3} x+e^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) d^{1/3}+e^{1/3} x\right)^2}}\right.$$
$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) d^{1/3}+e^{1/3} x}{\left(1+\sqrt{3}\right) d^{1/3}+e^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/$$
$$\left(15 \times 3^{1/4} d e^{7/3} \sqrt{\frac{d^{1/3}\left(d^{1/3}+e^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) d^{1/3}+e^{1/3} x\right)^2}} \sqrt{d+e x^3}\right)$$

Result (type 4, 197 leaves):

$$\left(2\left(3(-e)^{1/3} x\left(5 e(-b d+a e)+c d\left(8 d+3 e x^3\right)\right)-\right.\right.$$
$$i\, 3^{3/4} d^{1/3}\left(16 c d^2-5 e(2 b d+a e)\right) \sqrt{(-1)^{5/6}\left(-1+\frac{(-e)^{1/3} x}{d^{1/3}}\right)} \sqrt{1+\frac{(-e)^{1/3} x}{d^{1/3}}+\frac{(-e)^{2/3} x^2}{d^{2/3}}}$$
$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-e)^{1/3} x}{d^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right) \Big/\left(45 d(-e)^{7/3} \sqrt{d+e x^3}\right)$$

Problem 40: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b x^3+c x^6}{\left(d+e x^3\right)^{5/2}} \, dx$$

Optimal (type 4, 309 leaves, 3 steps):

$$\frac{2\left(c d^2-b d e+a e^2\right) x}{9 d e^2\left(d+e x^3\right)^{3/2}}-\frac{2\left(11 c d^2-2 b d e-7 a e^2\right) x}{27 d^2 e^2 \sqrt{d+e x^3}}+\left(2 \sqrt{2+\sqrt{3}}\left(16 c d^2+e(2 b d+7 a e)\right)\left(d^{1/3}+e^{1/3} x\right) \sqrt{\frac{d^{2/3}-d^{1/3} e^{1/3} x+e^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) d^{1/3}+e^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) d^{1/3}+e^{1/3} x}{\left(1+\sqrt{3}\right) d^{1/3}+e^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/ \left(27 \times 3^{1/4} d^2 e^{7/3} \sqrt{\frac{d^{1/3}\left(d^{1/3}+e^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) d^{1/3}+e^{1/3} x\right)^2}} \sqrt{d+e x^3}\right)$$

Result (type 4, 224 leaves):

$$\left(2\left(3(-e)^{1/3} x\left(-c d^2\left(8 d+11 e x^3\right)+e\left(-b d\left(d-2 e x^3\right)+a e\left(10 d+7 e x^3\right)\right)\right)+i\, 3^{3/4} d^{1/3}\left(16 c d^2+e(2 b d+7 a e)\right) \sqrt{(-1)^{5/6}\left(-1+\frac{(-e)^{1/3} x}{d^{1/3}}\right)} \sqrt{1+\frac{(-e)^{1/3} x}{d^{1/3}}+\frac{(-e)^{2/3} x^2}{d^{2/3}}}\left(d+e x^3\right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-e)^{1/3} x}{d^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right) \Big/ \left(81 d^2(-e)^{7/3}\left(d+e x^3\right)^{3/2}\right)$$

Problem 41: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b x^3+c x^6}{\left(d+e x^3\right)^{7/2}} \, dx$$

Optimal (type 4, 349 leaves, 4 steps):

$$\frac{2\left(c d^2-b d e+a e^2\right) x}{15 d e^2\left(d+e x^3\right)^{5/2}}-\frac{2\left(17 c d^2-2 b d e-13 a e^2\right) x}{135 d^2 e^2\left(d+e x^3\right)^{3/2}}+$$
$$\frac{2\left(16 c d^2+14 b d e+91 a e^2\right) x}{405 d^3 e^2 \sqrt{d+e x^3}}+\left(2 \sqrt{2+\sqrt{3}}\left(16 c d^2+14 b d e+91 a e^2\right)\left(d^{1/3}+e^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{d^{2/3}-d^{1/3} e^{1/3} x+e^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) d^{1/3}+e^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) d^{1/3}+e^{1/3} x}{\left(1+\sqrt{3}\right) d^{1/3}+e^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/$$
$$\left(405 \times 3^{1/4} d^3 e^{7/3} \sqrt{\frac{d^{1/3}\left(d^{1/3}+e^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) d^{1/3}+e^{1/3} x\right)^2}} \sqrt{d+e x^3}\right)$$

Result (type 4, 262 leaves):

$$\frac{1}{1215 d^3 (-e)^{7/3}\left(d+e x^3\right)^{5/2}}$$
$$2\left(3 (-e)^{1/3} x\left(27 d^2\left(c d^2+e(-b d+a e)\right)-3 d\left(17 c d^2-e(2 b d+13 a e)\right)\left(d+e x^3\right)+\right.\right.$$
$$\left.\left(16 c d^2+7 e(2 b d+13 a e)\right)\left(d+e x^3\right)^2\right)+i\, 3^{3/4} d^{1/3}\left(16 c d^2+7 e(2 b d+13 a e)\right)$$
$$\sqrt{(-1)^{5/6}\left(-1+\frac{(-e)^{1/3} x}{d^{1/3}}\right)} \sqrt{1+\frac{(-e)^{1/3} x}{d^{1/3}}+\frac{(-e)^{2/3} x^2}{d^{2/3}}}\left(d+e x^3\right)^2$$
$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-e)^{1/3} x}{d^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 42: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b x^3+c x^6}{\left(d+e x^3\right)^{9/2}} \, dx$$

Optimal (type 4, 389 leaves, 5 steps):

$$\frac{2\left(c d^2-b d e+a e^2\right) x}{21 d e^2\left(d+e x^3\right)^{7/2}}-\frac{2\left(23 c d^2-2 b d e-19 a e^2\right) x}{315 d^2 e^2\left(d+e x^3\right)^{5/2}}+\frac{2\left(16 c d^2+26 b d e+247 a e^2\right) x}{2835 d^3 e^2\left(d+e x^3\right)^{3/2}}+\frac{2\left(16 c d^2+26 b d e+247 a e^2\right) x}{1215 d^4 e^2\sqrt{d+e x^3}}+$$

$$\left(2\sqrt{2+\sqrt{3}}\left(16 c d^2+26 b d e+247 a e^2\right)\left(d^{1/3}+e^{1/3} x\right)\sqrt{\frac{d^{2/3}-d^{1/3} e^{1/3} x+e^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) d^{1/3}+e^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) d^{1/3}+e^{1/3} x}{\left(1+\sqrt{3}\right) d^{1/3}+e^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(1215\times 3^{1/4} d^4 e^{7/3}\sqrt{\frac{d^{1/3}\left(d^{1/3}+e^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) d^{1/3}+e^{1/3} x\right)^2}}\sqrt{d+e x^3}\right)$$

Result (type 4, 296 leaves):

$$\frac{1}{25515 d^4 (-e)^{7/3}\left(d+e x^3\right)^{7/2}}$$

$$2\left(3 (-e)^{1/3} x\left(405 d^3\left(c d^2+e(-b d+a e)\right)-27 d^2\left(23 c d^2-e(2 b d+19 a e)\right)\left(d+e x^3\right)+\right.\right.$$

$$\left.3 d\left(16 c d^2+13 e(2 b d+19 a e)\right)\left(d+e x^3\right)^2+7\left(16 c d^2+13 e(2 b d+19 a e)\right)\left(d+e x^3\right)^3\right)+$$

$$7\,i\,3^{3/4} d^{1/3}\left(16 c d^2+13 e(2 b d+19 a e)\right)\sqrt{(-1)^{5/6}\left(-1+\frac{(-e)^{1/3} x}{d^{1/3}}\right)}$$

$$\sqrt{1+\frac{(-e)^{1/3} x}{d^{1/3}}+\frac{(-e)^{2/3} x^2}{d^{2/3}}}\left(d+e x^3\right)^3$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-e)^{1/3} x}{d^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 43: Result is not expressed in closed-form.

$$\int\frac{x^4\left(d+e x^4\right)}{a+b x^4+c x^8}\,dx$$

Optimal (type 3, 433 leaves, 8 steps):

$$\frac{e\,x}{c}-\frac{\left(c\,d-b\,e+\frac{b\,c\,d-b^2\,e+2\,a\,c\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\,x}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2\times 2^{1/4}\,c^{5/4}\left(-b-\sqrt{b^2-4\,a\,c}\right)^{3/4}}-$$

$$\frac{\left(c\,d-b\,e-\frac{b\,c\,d-b^2\,e+2\,a\,c\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\,x}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2\times 2^{1/4}\,c^{5/4}\left(-b+\sqrt{b^2-4\,a\,c}\right)^{3/4}}-$$

$$\frac{\left(c\,d-b\,e+\frac{b\,c\,d-b^2\,e+2\,a\,c\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\,x}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2\times 2^{1/4}\,c^{5/4}\left(-b-\sqrt{b^2-4\,a\,c}\right)^{3/4}}-$$

$$\frac{\left(c\,d-b\,e-\frac{b\,c\,d-b^2\,e+2\,a\,c\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\,x}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2\times 2^{1/4}\,c^{5/4}\left(-b+\sqrt{b^2-4\,a\,c}\right)^{3/4}}$$

Result (type 7, 88 leaves):

$$\frac{e\,x}{c}-\frac{\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\ \frac{a\,e\,\text{Log}[x-\#1]-c\,d\,\text{Log}[x-\#1]\,\#1^4+b\,e\,\text{Log}[x-\#1]\,\#1^4}{b\,\#1^3+2\,c\,\#1^7}\,\&\right]}{4\,c}$$

Problem 45: Result is not expressed in closed-form.

$$\int\frac{x^2\left(d+e\,x^4\right)}{a+b\,x^4+c\,x^8}\,dx$$

Optimal (type 3, 375 leaves, 7 steps):

$$\frac{\left(e-\frac{2\,c\,d-b\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\,x}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2\times 2^{3/4}\,c^{3/4}\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}+\frac{\left(e+\frac{2\,c\,d-b\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTan}\left[\frac{2^{1/4}\,c^{1/4}\,x}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2\times 2^{3/4}\,c^{3/4}\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}-$$

$$\frac{\left(e-\frac{2\,c\,d-b\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\,x}{\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2\times 2^{3/4}\,c^{3/4}\left(-b-\sqrt{b^2-4\,a\,c}\right)^{1/4}}-\frac{\left(e+\frac{2\,c\,d-b\,e}{\sqrt{b^2-4\,a\,c}}\right)\text{ArcTanh}\left[\frac{2^{1/4}\,c^{1/4}\,x}{\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}\right]}{2\times 2^{3/4}\,c^{3/4}\left(-b+\sqrt{b^2-4\,a\,c}\right)^{1/4}}$$

Result (type 7, 59 leaves):

$$\frac{1}{4}\,\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\ \frac{d\,\text{Log}[x-\#1]+e\,\text{Log}[x-\#1]\,\#1^4}{b\,\#1+2\,c\,\#1^5}\,\&\right]$$

Problem 47: Result is not expressed in closed-form.

$$\int \frac{d+e x^4}{a+b x^4+c x^8} \, dx$$

Optimal (type 3, 375 leaves, 7 steps):

$$-\frac{\left(e-\frac{2 c d-b e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{2^{1/4} c^{1/4} x}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{2\times 2^{1/4} c^{1/4} \left(-b-\sqrt{b^2-4 a c}\right)^{3/4}}-\frac{\left(e+\frac{2 c d-b e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{2^{1/4} c^{1/4} x}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{2\times 2^{1/4} c^{1/4} \left(-b+\sqrt{b^2-4 a c}\right)^{3/4}}-\frac{\left(e-\frac{2 c d-b e}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} x}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{2\times 2^{1/4} c^{1/4} \left(-b-\sqrt{b^2-4 a c}\right)^{3/4}}-\frac{\left(e+\frac{2 c d-b e}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} x}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{2\times 2^{1/4} c^{1/4} \left(-b+\sqrt{b^2-4 a c}\right)^{3/4}}$$

Result (type 7, 61 leaves):

$$\frac{1}{4} \text{RootSum}\left[a+b \#1^4+c \#1^8\&,\frac{d \text{Log}[x-\#1]+e \text{Log}[x-\#1] \#1^4}{b \#1^3+2 c \#1^7}\&\right]$$

Problem 48: Result is not expressed in closed-form.

$$\int \frac{d+e x^4}{x \left(a+b x^4+c x^8\right)} \, dx$$

Optimal (type 3, 78 leaves, 7 steps):

$$\frac{(b d-2 a e) \text{ArcTanh}\left[\frac{b+2 c x^4}{\sqrt{b^2-4 a c}}\right]}{4 a \sqrt{b^2-4 a c}}+\frac{d \text{Log}[x]}{a}-\frac{d \text{Log}\left[a+b x^4+c x^8\right]}{8 a}$$

Result (type 7, 80 leaves):

$$\frac{d \text{Log}[x]}{a}-\frac{\text{RootSum}\left[a+b \#1^4+c \#1^8\&,\frac{b d \text{Log}[x-\#1]-a e \text{Log}[x-\#1]+c d \text{Log}[x-\#1] \#1^4}{b+2 c \#1^4}\&\right]}{4 a}$$

Problem 49: Result is not expressed in closed-form.

$$\int \frac{d+e x^4}{x^2 \left(a+b x^4+c x^8\right)} \, dx$$

Optimal (type 3, 392 leaves, 8 steps):

$$-\frac{d}{a x}-\frac{c^{1/4}\left(d-\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right)\text{ArcTan}\left[\frac{2^{1/4} c^{1/4} x}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{2\times 2^{3/4} a\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}-\frac{c^{1/4}\left(d+\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right)\text{ArcTan}\left[\frac{2^{1/4} c^{1/4} x}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{2\times 2^{3/4} a\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}+\frac{c^{1/4}\left(d-\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right)\text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} x}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{2\times 2^{3/4} a\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}+\frac{c^{1/4}\left(d+\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right)\text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} x}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{2\times 2^{3/4} a\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}$$

Result (type 7, 85 leaves):

$$-\frac{d}{a x}-\frac{\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\ \frac{b d \text{Log}[x-\#1]-a e \text{Log}[x-\#1]+c d \text{Log}[x-\#1]\,\#1^4}{b\,\#1+2 c\,\#1^5}\,\&\right]}{4 a}$$

Problem 50: Result is not expressed in closed-form.

$$\int \frac{d+e x^4}{x^3\left(a+b x^4+c x^8\right)}\,dx$$

Optimal (type 3, 199 leaves, 5 steps):

$$-\frac{d}{2 a x^2}-\frac{\sqrt{c}\left(d+\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right)\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4 a c}}}\right]}{2\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4 a c}}}-\frac{\sqrt{c}\left(d-\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right)\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b+\sqrt{b^2-4 a c}}}\right]}{2\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4 a c}}}$$

Result (type 7, 89 leaves):

$$-\frac{d}{2 a x^2}-\frac{\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\ \frac{b d \text{Log}[x-\#1]-a e \text{Log}[x-\#1]+c d \text{Log}[x-\#1]\,\#1^4}{b\,\#1^2+2 c\,\#1^6}\,\&\right]}{4 a}$$

Problem 51: Result is not expressed in closed-form.

$$\int \frac{d+e x^4}{x^4\left(a+b x^4+c x^8\right)}\,dx$$

Optimal (type 3, 394 leaves, 8 steps):

$$-\frac{d}{3 a x^3}+\frac{c^{3/4}\left(d-\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right)\text{ArcTan}\left[\frac{2^{1/4} c^{1/4} x}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{2\times 2^{1/4} a\left(-b-\sqrt{b^2-4 a c}\right)^{3/4}}+\frac{c^{3/4}\left(d+\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right)\text{ArcTan}\left[\frac{2^{1/4} c^{1/4} x}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{2\times 2^{1/4} a\left(-b+\sqrt{b^2-4 a c}\right)^{3/4}}+\frac{c^{3/4}\left(d-\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right)\text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} x}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{2\times 2^{1/4} a\left(-b-\sqrt{b^2-4 a c}\right)^{3/4}}+\frac{c^{3/4}\left(d+\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right)\text{ArcTanh}\left[\frac{2^{1/4} c^{1/4} x}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{2\times 2^{1/4} a\left(-b+\sqrt{b^2-4 a c}\right)^{3/4}}$$

Result (type 7, 86 leaves):

$$-\frac{1}{12\,a}\left(\frac{4\,d}{x^3}+3\,\text{RootSum}\left[a+b\,\#1^4+c\,\#1^8\,\&,\,\frac{b\,d\,\text{Log}[x-\#1]-a\,e\,\text{Log}[x-\#1]+c\,d\,\text{Log}[x-\#1]\,\#1^4}{b\,\#1^3+2\,c\,\#1^7}\,\&\right]\right)$$

Problem 52: Result is not expressed in closed-form.

$$\int\frac{x^4\left(1-x^4\right)}{1-x^4+x^8}\,dx$$

Optimal (type 3, 278 leaves, 20 steps):

$$-x-\frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{3}}-2\,x}{\sqrt{2+\sqrt{3}}}\right]}{2\sqrt{6}}-\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{3}}-2\,x}{\sqrt{2-\sqrt{3}}}\right]}{2\sqrt{6}}+\frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{3}}+2\,x}{\sqrt{2+\sqrt{3}}}\right]}{2\sqrt{6}}+\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{3}}+2\,x}{\sqrt{2-\sqrt{3}}}\right]}{2\sqrt{6}}-\frac{\text{Log}\left[1-\sqrt{2-\sqrt{3}}\,x+x^2\right]}{4\sqrt{6}}+\frac{\text{Log}\left[1+\sqrt{2-\sqrt{3}}\,x+x^2\right]}{4\sqrt{6}}-\frac{\text{Log}\left[1-\sqrt{2+\sqrt{3}}\,x+x^2\right]}{4\sqrt{6}}+\frac{\text{Log}\left[1+\sqrt{2+\sqrt{3}}\,x+x^2\right]}{4\sqrt{6}}$$

Result (type 7, 46 leaves):

$$-x+\frac{1}{4}\,\text{RootSum}\left[1-\#1^4+\#1^8\,\&,\,\frac{\text{Log}[x-\#1]}{-\#1^3+2\,\#1^7}\,\&\right]$$

Problem 54: Result is not expressed in closed-form.

$$\int\frac{x^2\left(1-x^4\right)}{1-x^4+x^8}\,dx$$

Optimal (type 3, 355 leaves, 21 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{3}}-2\,x}{\sqrt{2+\sqrt{3}}}\right]}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{3}}-2\,x}{\sqrt{2-\sqrt{3}}}\right]}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{3}}+2\,x}{\sqrt{2+\sqrt{3}}}\right]}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{3}}+2\,x}{\sqrt{2-\sqrt{3}}}\right]}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\text{Log}\left[1-\sqrt{2-\sqrt{3}}\,x+x^2\right]-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\text{Log}\left[1+\sqrt{2-\sqrt{3}}\,x+x^2\right]-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\text{Log}\left[1-\sqrt{2+\sqrt{3}}\,x+x^2\right]+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\text{Log}\left[1+\sqrt{2+\sqrt{3}}\,x+x^2\right]$$

Result (type 7, 55 leaves):

$$-\frac{1}{4}\,\text{RootSum}\left[1-\#1^4+\#1^8\,\&,\,\frac{-\text{Log}[x-\#1]+\text{Log}[x-\#1]\,\#1^4}{-\#1+2\,\#1^5}\,\&\right]$$

Problem 56: Result is not expressed in closed-form.

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx$$

Optimal (type 3, 355 leaves, 19 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right]}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right]}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right]}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right]}{4\sqrt{3\left(2+\sqrt{3}\right)}}+$$
$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\text{Log}\left[1-\sqrt{2-\sqrt{3}}\,x+x^2\right]-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\text{Log}\left[1+\sqrt{2-\sqrt{3}}\,x+x^2\right]-$$
$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\text{Log}\left[1-\sqrt{2+\sqrt{3}}\,x+x^2\right]+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\text{Log}\left[1+\sqrt{2+\sqrt{3}}\,x+x^2\right]$$

Result (type 7, 57 leaves):

$$-\frac{1}{4}\,\text{RootSum}\left[1-\#1^4+\#1^8\,\&,\ \frac{-\text{Log}[x-\#1]+\text{Log}[x-\#1]\,\#1^4}{-\#1^3+2\,\#1^7}\,\&\right]$$

Problem 57: Result is not expressed in closed-form.

$$\int \frac{1-x^4}{x\left(1-x^4+x^8\right)}\,dx$$

Optimal (type 3, 41 leaves, 7 steps):

$$\frac{\text{ArcTan}\left[\frac{1-2x^4}{\sqrt{3}}\right]}{4\sqrt{3}}+\text{Log}[x]-\frac{1}{8}\,\text{Log}\left[1-x^4+x^8\right]$$

Result (type 7, 44 leaves):

$$\text{Log}[x]-\frac{1}{4}\,\text{RootSum}\left[1-\#1^4+\#1^8\,\&,\ \frac{\text{Log}[x-\#1]\,\#1^4}{-1+2\,\#1^4}\,\&\right]$$

Problem 58: Result is not expressed in closed-form.

$$\int \frac{1-x^4}{x^2\left(1-x^4+x^8\right)}\,dx$$

Optimal (type 3, 280 leaves, 20 steps):

$$-\frac{1}{x}+\frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right]}{2\sqrt{6}}+\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right]}{2\sqrt{6}}-\frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right]}{2\sqrt{6}}-\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right]}{2\sqrt{6}}-\frac{\text{Log}\left[1-\sqrt{2-\sqrt{3}}\,x+x^2\right]}{4\sqrt{6}}+\frac{\text{Log}\left[1+\sqrt{2-\sqrt{3}}\,x+x^2\right]}{4\sqrt{6}}-\frac{\text{Log}\left[1-\sqrt{2+\sqrt{3}}\,x+x^2\right]}{4\sqrt{6}}+\frac{\text{Log}\left[1+\sqrt{2+\sqrt{3}}\,x+x^2\right]}{4\sqrt{6}}$$

Result (type 7, 47 leaves):

$$-\frac{1}{x}-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\,\&,\frac{\text{Log}[x-\#1]\,\#1^3}{-1+2\,\#1^4}\,\&\right]$$

Problem 59: Result is not expressed in closed-form.

$$\int\frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx$$

Optimal (type 3, 89 leaves, 11 steps):

$$-\frac{1}{2x^2}+\frac{1}{4}\text{ArcTan}\left[\sqrt{3}-2x^2\right]-\frac{1}{4}\text{ArcTan}\left[\sqrt{3}+2x^2\right]-\frac{\text{Log}\left[1-\sqrt{3}\,x^2+x^4\right]}{8\sqrt{3}}+\frac{\text{Log}\left[1+\sqrt{3}\,x^2+x^4\right]}{8\sqrt{3}}$$

Result (type 7, 49 leaves):

$$-\frac{1}{2x^2}-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\,\&,\frac{\text{Log}[x-\#1]\,\#1^2}{-1+2\,\#1^4}\,\&\right]$$

Problem 60: Result is not expressed in closed-form.

$$\int\frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx$$

Optimal (type 3, 370 leaves, 21 steps):

$$-\frac{1}{3x^3}-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\ \text{ArcTan}\left[\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right]+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\ \text{ArcTan}\left[\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right]+$$
$$\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\ \text{ArcTan}\left[\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right]-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\ \text{ArcTan}\left[\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right]+$$
$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\ \text{Log}\left[1-\sqrt{2-\sqrt{3}}\ x+x^2\right]-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\ \text{Log}\left[1+\sqrt{2-\sqrt{3}}\ x+x^2\right]-$$
$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\ \text{Log}\left[1-\sqrt{2+\sqrt{3}}\ x+x^2\right]+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\ \text{Log}\left[1+\sqrt{2+\sqrt{3}}\ x+x^2\right]$$

Result (type 7, 47 leaves):

$$-\frac{1}{3x^3}-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\ \&,\ \frac{\text{Log}[x-\#1]\ \#1}{-1+2\ \#1^4}\ \&\right]$$

Problem 79: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\ x^4\sqrt{d+e\,x}\ dx$$

Optimal (type 4, 981 leaves, 11 steps):

$$-\frac{1}{3465\,a^4\,e^4}2\left(187\,a^4\,d^4+64\,b^4\,e^4+4\,a\,b^2\,e^3\left(7\,b\,d-69\,c\,e\right)-4\,a^3\,d^2\,e\left(2\,b\,d+3\,c\,e\right)+\right.$$
$$\left.3\,a^2\,e^2\left(3\,b^2\,d^2-29\,b\,c\,d\,e+50\,c^2\,e^2\right)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\ x\sqrt{d+e\,x}+\frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\ x^5\sqrt{d+e\,x}+$$
$$\frac{1}{3465\,a^3\,e^4}2\left(233\,a^3\,d^3+48\,b^3\,e^3+a\,b\,e^2\left(67\,b\,d-157\,c\,e\right)+4\,a^2\,d\,e\left(18\,b\,d-37\,c\,e\right)\right)$$
$$\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\ x\left(d+e\,x\right)^{3/2}-$$
$$\frac{2\left(29\,a^2\,d^2+8\,b^2\,e^2+a\,e\left(19\,b\,d-18\,c\,e\right)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\ x\left(d+e\,x\right)^{5/2}}{693\,a^2\,e^4}+$$
$$\frac{2\left(a\,d+b\,e\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\ x\left(d+e\,x\right)^{7/2}}{99\,a\,e^4}+$$

$$\left(\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(128\,a^5\,d^5+128\,b^5\,e^5-4\,a^4\,d^3\,e\,\left(14\,b\,d-27\,c\,e\right)-8\,a\,b^3\,e^4\,\left(7\,b\,d+87\,c\,e\right)-a^2\,b\,e^3\,\left(37\,b^2\,d^2-258\,b\,c\,d\,e-771\,c^2\,e^2\right)-a^3\,d\,e^2\,\left(37\,b^2\,d^2-135\,b\,c\,d\,e+156\,c^2\,e^2\right)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\;x\,\sqrt{d+e\,x}\,\sqrt{-\frac{a\,\left(c+b\,x+a\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,a\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\;e}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/\left(3465\,a^5\,e^5\,\sqrt{\frac{a\,\left(d+e\,x\right)}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\;\left(c+b\,x+a\,x^2\right)\right)-$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(a\,d^2-e\,\left(b\,d-c\,e\right)\right)\,\left(128\,a^4\,d^4-64\,b^4\,e^4-4\,a\,b^2\,e^3\,\left(7\,b\,d-69\,c\,e\right)+4\,a^3\,d^2\,e\,\left(2\,b\,d+3\,c\,e\right)-3\,a^2\,e^2\,\left(3\,b^2\,d^2-29\,b\,c\,d\,e+50\,c^2\,e^2\right)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\;x\,\sqrt{\frac{a\,\left(d+e\,x\right)}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{a\,\left(c+b\,x+a\,x^2\right)}{b^2-4\,a\,c}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,a\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\;e}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/\left(3465\,a^5\,e^5\,\sqrt{d+e\,x}\;\left(c+b\,x+a\,x^2\right)\right)$$

Result (type 4, 10 904 leaves):

$$x\,\sqrt{d+e\,x}\,\left(-\frac{1}{3465\,a^4\,e^4}4\,\left(32\,a^4\,d^4-10\,a^3\,b\,d^3\,e-9\,a^2\,b^2\,d^2\,e^2+23\,a^3\,c\,d^2\,e^2-10\,a\,b^3\,d\,e^3+35\,a^2\,b\,c\,d\,e^3+\right.\right.$$

$$32 b^4 e^4 - 138 a b^2 c e^4 + 75 a^2 c^2 e^4\Big) - \frac{1}{3465 a^3 e^3}$$

$$2 \left(-48 a^3 d^3 + 13 a^2 b d^2 e + 13 a b^2 d e^2 - 32 a^2 c d e^2 - 48 b^3 e^3 + 157 a b c e^3\right) x +$$

$$\left.\frac{4 \left(-4 a^2 d^2 + a b d e - 4 b^2 e^2 + 9 a c e^2\right) x^2}{693 a^2 e^2} + \frac{2 \left(a d + b e\right) x^3}{99 a e} + \frac{2 x^4}{11}\right)$$

$$\sqrt{a + \frac{c + b x}{x^2}} + \frac{1}{3465 a^4 e^6 \sqrt{c + b x + a x^2}} 2 x \sqrt{a + \frac{c + b x}{x^2}}$$

$$\Bigg(\Bigg(\Big(128 a^5 d^5 - 56 a^4 b d^4 e - 37 a^3 b^2 d^3 e^2 + 108 a^4 c d^3 e^2 - 37 a^2 b^3 d^2 e^3 + 135 a^3 b c d^2 e^3 -$$

$$56 a b^4 d e^4 + 258 a^2 b^2 c d e^4 - 156 a^3 c^2 d e^4 + 128 b^5 e^5 - 696 a b^3 c e^5 + 771 a^2 b c^2 e^5\Big)$$

$$\left(d + e x\right)^{3/2} \left(a + \frac{a d^2}{\left(d + e x\right)^2} - \frac{b d e}{\left(d + e x\right)^2} + \frac{c e^2}{\left(d + e x\right)^2} - \frac{2 a d}{d + e x} + \frac{b e}{d + e x}\right)\Bigg) \Big/$$

$$\left(a \sqrt{\frac{\left(d + e x\right)^2 \left(a \left(-1 + \frac{d}{d+e x}\right)^2 + \frac{e \left(b - \frac{b d}{d+e x} + \frac{c e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right) - \frac{1}{a \sqrt{\frac{(d+e x)^2 \left(a \left(-1+\frac{d}{d+e x}\right)^2 + \frac{e \left(b-\frac{b d}{d+e x}+\frac{c e}{d+e x}\right)}{d+e x}\right)}{e^2}}}$$

$$\left(a d^2 - b d e + c e^2\right) \left(d + e x\right) \sqrt{a + \frac{a d^2}{\left(d + e x\right)^2} - \frac{b d e}{\left(d + e x\right)^2} + \frac{c e^2}{\left(d + e x\right)^2} - \frac{2 a d}{d + e x} + \frac{b e}{d + e x}} \Bigg(\Bigg(32 i$$

$$\sqrt{2} a^5 d^5 \left(2 a d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(a d^2 - b d e + c e^2\right)}{\left(2 a d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) \left(d + e x\right)}}$$

$$\sqrt{1 - \frac{2 \left(a d^2 - b d e + c e^2\right)}{\left(2 a d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \left(d + e x\right)}}$$

$$\Bigg(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 a d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 a d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] -$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\right.$$

$$\left.\left.\left.\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/\left(\left(a d^2-b d e+c e^2\right)\right.$$

$$\left.\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)-\left(14\, i\,\sqrt{2}\right.$$

$$a^4 b d^4 e\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\right.$$

$$\left.\left.\left.\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/\left(\left(a d^2-b d e+c e^2\right)\right.$$

$$\left.\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)-\left(37\, i\, a^3\right.$$

$$b^2 d^3 e^2\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\left(a d^2-b d e+c e^2\right)\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)+\Bigg(27\,i\sqrt{2}\,a^4 c d^3 e^2$$

$$\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\right.$$

$$\left.\left.\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg)\Bigg/\Bigg(\left(a d^2-b d e+c e^2\right)$$

$$\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\Bigg)-\Bigg(37\,i\,a^2$$

$$b^3 d^2 e^3\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \Bigg(\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[$$

$$\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big],\ \frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]-\text{EllipticF}\Big[i$$

$$\text{ArcSinh}\Big[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big],\ \frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]\Bigg)\Bigg)\Bigg/$$

$$\Bigg(2\sqrt{2}\left(a d^2-b d e+c e^2\right)\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}$$

$$\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\Bigg)+\Bigg(135\,i\,a^3 b c d^2 e^3$$

$$\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\ \Bigg(\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[$$

$$\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big],\ \frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]-\text{EllipticF}\Big[i$$

$$\text{ArcSinh}\Big[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big],\ \frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]\Bigg)\Bigg)\Big/$$

$$\Bigg(2\sqrt{2}\,\left(a d^2-b d e+c e^2\right)\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}$$

$$\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\Bigg)-\Bigg(14\,\mathrm{i}\,\sqrt{2}\,a b^4 d e^4$$

$$\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\Bigg(\text{EllipticE}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big],\ \frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]-$$

$$\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big],$$

$$\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]\Bigg)\Bigg)\Big/\Bigg(\left(a d^2-b d e+c e^2\right)$$

$$\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\Bigg)+\Bigg(129\,\mathrm{i}\,a^2$$

$$b^2 c d e^4\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(\sqrt{2}\left(a d^2-b d e+c e^2\right)\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)-\left(39\,\mathrm{i}\sqrt{2}\,a^3 c^2 d e^4\right.$$

$$\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\right.$$

$$\left.\left.\left.\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/\left(\left(a d^2-b d e+c e^2\right)\right.$$

$$\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\Bigg)+\Bigg(32\, i$$

$$\sqrt{2}\ b^5 e^5 \left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\Bigg(\text{EllipticE}\Big[i\, \text{ArcSinh}\Big[\frac{\sqrt{2}\ \sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big], \frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]-$$

$$\text{EllipticF}\Big[i\, \text{ArcSinh}\Big[\frac{\sqrt{2}\ \sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big],$$

$$\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]\Bigg)\Bigg) \Big/ \Bigg(\left(a d^2-b d e+c e^2\right)$$

$$\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\Bigg)-$$

$$\Bigg(174\, i \sqrt{2}\ a b^3 c e^5 \left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)$$

$$\sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/\left(\left(a d^2-b d e+c e^2\right)\right.$$

$$\left.\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)+\left(771\,\mathrm{i}\right.$$

$$a^2 b c^2 e^5\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/\left(2\sqrt{2}\left(a d^2-b d e+c e^2\right)\right.$$

$$\left.\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)+$$

$$\left(64\, i \sqrt{2}\, a^5 d^4 \sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)+$$

$$\left(4\, i \sqrt{2}\, a^4 b\, d^3 e \sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)-$$

$$\left(9\, i\, a^3 b^2 d^2 e^2 \sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)+$$

$$\left(6\,\mathrm{i}\sqrt{2}\,a^4 c\,d^2 e^2\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)-$$

$$\left(14\,\mathrm{i}\sqrt{2}\,a^2 b^3 d\,e^3\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)+$$

$$\left(87\,\mathrm{i}\,a^3\,b\,c\,d\,e^3\,\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}\right.$$

$$\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{a+\frac{a\,d^2-b\,d\,e+c\,e^2}{\left(d+e\,x\right)^2}+\frac{-2\,a\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(32\,\mathrm{i}\,\sqrt{2}\,a\,b^4\,e^4\,\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}\right.$$

$$\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{a+\frac{a\,d^2-b\,d\,e+c\,e^2}{\left(d+e\,x\right)^2}+\frac{-2\,a\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(138\,\mathrm{i}\,\sqrt{2}\,a^2\,b^2\,c\,e^4\,\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}\right.$$

$$\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)-$$

$$\left(75\, i\,\sqrt{2}\, a^3 c^2 e^4\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left.\left(\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)\right)\right)$$

Problem 80: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x^3\sqrt{d+e x}\,dx$$

Optimal (type 4, 778 leaves, 10 steps):

$$\frac{1}{315 a^3 e^3}2\left(19 a^3 d^3-6 a^2 c d e^2+8 b^3 e^3+3 a b e^2(b d-9 c e)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+e x}+$$

$$\frac{2}{9}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x^4\sqrt{d+e x}-\frac{4\left(8 a^2 d^2+3 b^2 e^2+a e(4 b d-7 c e)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\,(d+e x)^{3/2}}{315 a^2 e^3}+$$

$$\frac{2\,(a\,d+b\,e)\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\;x\,(d+e\,x)^{5/2}}{63\,a\,e^3}-$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(8\,a^4\,d^4+8\,b^4\,e^4-a^3\,d^2\,e\,(4\,b\,d-9\,c\,e)-4\,a\,b^2\,e^3\,(b\,d+9\,c\,e)-\right.\right.$$

$$\left.3\,a^2\,e^2\,\left(b^2\,d^2-5\,b\,c\,d\,e-7\,c^2\,e^2\right)\right)\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\;x\,\sqrt{d+e\,x}\,\sqrt{-\frac{a\,\left(c+b\,x+a\,x^2\right)}{b^2-4\,a\,c}}$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,a\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\;e}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/$$

$$\left(315\,a^4\,e^4\,\sqrt{\frac{a\,(d+e\,x)}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\;\left(c+b\,x+a\,x^2\right)\right)+$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(16\,a^3\,d^3+6\,a^2\,c\,d\,e^2-8\,b^3\,e^3-3\,a\,b\,e^2\,(b\,d-9\,c\,e)\right)\right.$$

$$\left(a\,d^2-e\,(b\,d-c\,e)\right)\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\;x\,\sqrt{\frac{a\,(d+e\,x)}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{a\,\left(c+b\,x+a\,x^2\right)}{b^2-4\,a\,c}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,a\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\;e}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/$$

$$\left(315\,a^4\,e^4\,\sqrt{d+e\,x}\,\left(c+b\,x+a\,x^2\right)\right)$$

Result (type 4, 7531 leaves) :

$$x\sqrt{d+e x}\left(-\frac{1}{315 a^3 e^3}2\left(-8 a^3 d^3+3 a^2 b d^2 e+3 a b^2 d e^2-8 a^2 c d e^2-8 b^3 e^3+27 a b c e^3\right)+\frac{4\left(-3 a^2 d^2+a b d e-3 b^2 e^2+7 a c e^2\right) x}{315 a^2 e^2}+\frac{2\left(a d+b e\right) x^2}{63 a e}+\frac{2 x^3}{9}\right)$$

$$\sqrt{a+\frac{c+b x}{x^2}}-\frac{1}{315 a^3 e^5\sqrt{c+b x+a x^2}}2 x\sqrt{a+\frac{c+b x}{x^2}}$$

$$\left(\left(2\left(8 a^4 d^4-4 a^3 b d^3 e-3 a^2 b^2 d^2 e^2+9 a^3 c d^2 e^2-4 a b^3 d e^3+15 a^2 b c d e^3+8 b^4 e^4-36 a b^2 c e^4+\right.\right.\right.$$

$$\left.\left.21 a^2 c^2 e^4\right)\left(d+e x\right)^{3/2}\left(a+\frac{a d^2}{\left(d+e x\right)^2}-\frac{b d e}{\left(d+e x\right)^2}+\frac{c e^2}{\left(d+e x\right)^2}-\frac{2 a d}{d+e x}+\frac{b e}{d+e x}\right)\right)\Big/$$

$$\left(a\sqrt{\frac{\left(d+e x\right)^2\left(a\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{c e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-\frac{1}{a\sqrt{\frac{(d+e x)^2\left(a\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{c e}{d+e x}\right)}{d+e x}\right)}{e^2}}}$$

$$\left(a d^2-b d e+c e^2\right)\left(d+e x\right)\sqrt{a+\frac{a d^2}{\left(d+e x\right)^2}-\frac{b d e}{\left(d+e x\right)^2}+\frac{c e^2}{\left(d+e x\right)^2}-\frac{2 a d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(\left(4\,\mathrm{i}\sqrt{2}\, a^4 d^4\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)\left(d+e x\right)}}\right.\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\left(d+e x\right)}}$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\right.$$

$$\left.\left.\left.\frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Bigg/\left(\left(a\,d^2-b\,d\,e+c\,e^2\right)\right.$$

$$\left.\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{a+\frac{a\,d^2-b\,d\,e+c\,e^2}{\left(d+e\,x\right)^2}+\frac{-2\,a\,d+b\,e}{d+e\,x}}\right)-\left(2\,\mathrm{i}\,\sqrt{2}\right.$$

$$a^3\,b\,d^3\,e\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\left(d+e\,x\right)}}$$

$$\sqrt{1-\frac{2\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\left(d+e\,x\right)}}$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\ \frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\right.$$

$$\left.\left.\left.\frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Bigg/\left(\left(a\,d^2-b\,d\,e+c\,e^2\right)\right.$$

$$\left.\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\ \sqrt{a+\frac{a\,d^2-b\,d\,e+c\,e^2}{\left(d+e\,x\right)^2}+\frac{-2\,a\,d+b\,e}{d+e\,x}}\right)-\left(3\,\mathrm{i}\,a^2\right.$$

$$b^2\,d^2\,e^2\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\left(d+e\,x\right)}}$$

$$\sqrt{1-\frac{2\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\left(d+e\,x\right)}}\ \left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\right.\right.\right.$$

$$\frac{\sqrt{2}\ \sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Big],\ \frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Big]-\text{EllipticF}\Big[\mathbb{i}$$

$$\text{ArcSinh}\Big[\frac{\sqrt{2}\ \sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Big],\ \frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Big]\Bigg)\Bigg)\Bigg/$$

$$\Bigg(\sqrt{2}\ \left(a\,d^2-b\,d\,e+c\,e^2\right)\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}$$

$$\sqrt{a+\frac{a\,d^2-b\,d\,e+c\,e^2}{(d+e\,x)^2}+\frac{-2\,a\,d+b\,e}{d+e\,x}}\ \Bigg)+\Bigg(9\,\mathbb{i}\,a^3\,c\,d^2\,e^2$$

$$\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}$$

$$\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\ \Bigg(\text{EllipticE}\Big[\mathbb{i}\,\text{ArcSinh}\Big[$$

$$\frac{\sqrt{2}\ \sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Big],\ \frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Big]-\text{EllipticF}\Big[\mathbb{i}$$

$$\text{ArcSinh}\Big[\frac{\sqrt{2}\ \sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\Big],\ \frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\Big]\Bigg)\Bigg)\Bigg/$$

$$\Bigg(\sqrt{2}\ \left(a\,d^2-b\,d\,e+c\,e^2\right)\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}$$

$$\sqrt{a+\frac{a\,d^2-b\,d\,e+c\,e^2}{(d+e\,x)^2}+\frac{-2\,a\,d+b\,e}{d+e\,x}}\ \Bigg)-\Bigg(2\,\mathbb{i}\,\sqrt{2}\,a\,b^3\,d\,e^3$$

$$\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\left(\text{EllipticE}\left[\mathbb{i} \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathbb{i} \,\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\right.$$

$$\left.\left.\left.\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right) \Bigg/ \left(\left(a d^2-b d e+c e^2\right)\right.$$

$$\left.\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)+\left(15 \,\mathbb{i}\, a^2\right.$$

$$b c d e^3 \left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}} \left(\text{EllipticE}\left[\mathbb{i} \,\text{ArcSinh}\left[\right.\right.\right.$$

$$\left.\frac{\sqrt{2} \sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\text{EllipticF}\left[\mathbb{i}\right.$$

$$\left.\left.\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right) \Bigg/$$

$$\left(\sqrt{2}\,\left(a d^2-b d e+c e^2\right)\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\right.$$

$$\left.\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)+\left(4\,\mathrm{i}\,\sqrt{2}\,b^4 e^4\right.$$

$$\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\right.$$

$$\left.\left.\left.\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/\left(\left(a d^2-b d e+c e^2\right)\right.$$

$$\left.\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)-$$

$$\left(18\,\mathrm{i}\,\sqrt{2}\,a b^2 c e^4\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\right.$$

$$\left.\left.\left.\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Bigg/\left(\left(a d^2-b d e+c e^2\right)\right.$$

$$\left.\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\;\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)+$$

$$\left(21\,i\,a^2 c^2 e^4\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\right.$$

$$\left.\left.\left.\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Bigg/\left(\sqrt{2}\,\left(a d^2-b d e+c e^2\right)\right.$$

$$\left.\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\;\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)+$$

$$\left(8\,\mathrm{i}\,\sqrt{2}\,a^4\,d^3\,\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}\right.$$

$$\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{a+\frac{a\,d^2-b\,d\,e+c\,e^2}{\left(d+e\,x\right)^2}+\frac{-2\,a\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(3\,\mathrm{i}\,a^2\,b^2\,d\,e^2\,\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}\right.$$

$$\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{a+\frac{a\,d^2-b\,d\,e+c\,e^2}{\left(d+e\,x\right)^2}+\frac{-2\,a\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(3\,\mathrm{i}\,\sqrt{2}\,a^3\,c\,d\,e^2\,\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}\right.$$

$$\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}$$

$$\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)-$$

$$\left(4\,\text{i}\,\sqrt{2}\,a b^3 e^3\,\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left.\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)+$$

$$\left(27\,\text{i}\,a^2 b c e^3\,\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left.\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)\Bigg)\Bigg)$$

Problem 81: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x^2 \sqrt{d+e x}\, dx$$

Optimal (type 4, 636 leaves, 8 steps):

$$-\frac{1}{105 a^2 e^2}2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\sqrt{d+e x}\left(4 a^2 d^2+4 b^2 e^2-a e(2 b d-5 c e)-3 a e(a d-4 b e) x\right)+$$

$$\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\sqrt{d+e x}\left(c+b x+a x^2\right)}{7 a}+$$

$$\left(\sqrt{2}\sqrt{b^2-4 a c}\left(8 a^3 d^3+8 b^3 e^3-a^2 d e(5 b d-16 c e)-a b e^2(5 b d+29 c e)\right)\right.$$

$$\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\sqrt{d+e x}\sqrt{-\frac{a\left(c+b x+a x^2\right)}{b^2-4 a c}}$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 a x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\, e}{2 a d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Bigg/$$

$$\left(105 a^3 e^3\sqrt{\frac{a(d+e x)}{2 a d-\left(b+\sqrt{b^2-4 a c}\right) e}}\left(c+b x+a x^2\right)\right)-$$

$$\left(2\sqrt{2}\sqrt{b^2-4ac}\left(8a^2d^2-4b^2e^2-ae\left(bd-10ce\right)\right)\left(ad^2-e\left(bd-ce\right)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right.$$

$$\sqrt{\frac{a\left(d+ex\right)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a\left(c+bx+ax^2\right)}{b^2-4ac}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],\right.$$

$$\left.\left.-\frac{2\sqrt{b^2-4ac}\,e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right]\right)\Bigg/\left(105a^3e^3\sqrt{d+ex}\left(c+bx+ax^2\right)\right)$$

Result (type 4, 5350 leaves):

$$x\sqrt{d+ex}\left(\frac{4\left(-2a^2d^2+abde-2b^2e^2+5ace^2\right)}{105a^2e^2}+\frac{2\left(ad+be\right)x}{35ae}+\frac{2x^2}{7}\right)\sqrt{a+\frac{c+bx}{x^2}}+$$

$$\frac{1}{105a^2e^4\sqrt{c+bx+ax^2}}$$

$$2x\sqrt{a+\frac{c+bx}{x^2}}\left(\left(\left(8a^3d^3-5a^2bd^2e-5ab^2de^2+16a^2cde^2+8b^3e^3-29abce^3\right)\right.\right.$$

$$\left.\left.\left(d+ex\right)^{3/2}\left(a+\frac{ad^2}{\left(d+ex\right)^2}-\frac{bde}{\left(d+ex\right)^2}+\frac{ce^2}{\left(d+ex\right)^2}-\frac{2ad}{d+ex}+\frac{be}{d+ex}\right)\right)\right/$$

$$\left(a\sqrt{\frac{\left(d+ex\right)^2\left(a\left(-1+\frac{d}{d+ex}\right)^2+\frac{e\left(b-\frac{bd}{d+ex}+\frac{ce}{d+ex}\right)}{d+ex}\right)}{e^2}}\right)-\frac{1}{a\sqrt{\frac{\left(d+ex\right)^2\left(a\left(-1+\frac{d}{d+ex}\right)^2+\frac{e\left(b-\frac{bd}{d+ex}+\frac{ce}{d+ex}\right)}{d+ex}\right)}{e^2}}}$$

$$\left(ad^2-bde+ce^2\right)\left(d+ex\right)\sqrt{a+\frac{ad^2}{\left(d+ex\right)^2}-\frac{bde}{\left(d+ex\right)^2}+\frac{ce^2}{\left(d+ex\right)^2}-\frac{2ad}{d+ex}+\frac{be}{d+ex}}$$

$$\left(\left(2\,i\,\sqrt{2}\,a^3\,d^3\,\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}\right.\right.$$

$$\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\right.$$

$$\left.\left.\left.\frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Big/\left(\left(a\,d^2-b\,d\,e+c\,e^2\right)\right.$$

$$\left.\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{a+\frac{a\,d^2-b\,d\,e+c\,e^2}{\left(d+e\,x\right)^2}+\frac{-2\,a\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(5\,i\,a^2\,b\,d^2\,e\,\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}\right.$$

$$\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\right.$$

$$\left.\left.\left.\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/\left(2 \sqrt{2} \left(a d^2-b d e+c e^2\right)\right.$$

$$\left.\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)-$$

$$\left(5 i a b^2 d e^2 \left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}\right.$$

$$\sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}} \left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\right.\right.\right.$$

$$\left.\left.\frac{\sqrt{2} \sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\text{EllipticF}\left[i\right.$$

$$\left.\left.\left.\text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2 \sqrt{2} \left(a d^2-b d e+c e^2\right) \sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\right.$$

$$\left.\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)+\left(4 i \sqrt{2} a^2 c d e^2\right.$$

$$\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\right.$$

$$\left.\left.\left.\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Bigg/\left(\left(a d^2-b d e+c e^2\right)\right.$$

$$\left.\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)+\left(2\,\mathbb{i}\right.$$

$$\sqrt{2}\, b^3 e^3\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\right.$$

$$\left.\left.\left.\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Bigg/\left(\left(a d^2-b d e+c e^2\right)\right.$$

$$\left.\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)-$$

$$\left(29\, i\, a\, b\, c\, e^3 \left(2\, a\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}\right) \sqrt{1 - \frac{2\left(a\, d^2 - b\, d\, e + c\, e^2\right)}{\left(2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}\right)(d + e\, x)}}\right.$$

$$\sqrt{1 - \frac{2\left(a\, d^2 - b\, d\, e + c\, e^2\right)}{\left(2\, a\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}\right)(d + e\, x)}}$$

$$\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a\, d^2 - b\, d\, e + c\, e^2}{2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}}}{\sqrt{d + e\, x}}\right], \frac{2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}{2\, a\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}\right] -\right.$$

$$\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a\, d^2 - b\, d\, e + c\, e^2}{2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}}}{\sqrt{d + e\, x}}\right],\right.$$

$$\left.\left.\left.\frac{2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}{2\, a\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}\right]\right)\right) \Big/ \left(2\sqrt{2}\left(a\, d^2 - b\, d\, e + c\, e^2\right)\right.$$

$$\left.\sqrt{-\frac{a\, d^2 - b\, d\, e + c\, e^2}{2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}}\sqrt{a + \frac{a\, d^2 - b\, d\, e + c\, e^2}{(d + e\, x)^2} + \frac{-2\, a\, d + b\, e}{d + e\, x}}\right) +$$

$$\left(4\, i\sqrt{2}\, a^3 d^2 \sqrt{1 - \frac{2\left(a\, d^2 - b\, d\, e + c\, e^2\right)}{\left(2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}\right)(d + e\, x)}}\right.$$

$$\sqrt{1 - \frac{2\left(a\, d^2 - b\, d\, e + c\, e^2\right)}{\left(2\, a\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}\right)(d + e\, x)}}$$

$$\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a\, d^2 - b\, d\, e + c\, e^2}{2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}}}{\sqrt{d + e\, x}}\right], \frac{2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}{2\, a\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{a\, d^2 - b\, d\, e + c\, e^2}{2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}}\sqrt{a + \frac{a\, d^2 - b\, d\, e + c\, e^2}{(d + e\, x)^2} + \frac{-2\, a\, d + b\, e}{d + e\, x}}\right) -$$

$$\left(\mathbb{i}\, a^2\, b\, d\, e\, \sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{a+\frac{a\,d^2-b\,d\,e+c\,e^2}{(d+e\,x)^2}+\frac{-2\,a\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(2\,\mathbb{i}\,\sqrt{2}\,a\,b^2\,e^2\,\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{a+\frac{a\,d^2-b\,d\,e+c\,e^2}{(d+e\,x)^2}+\frac{-2\,a\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(5\,\mathbb{i}\,\sqrt{2}\,a^2\,c\,e^2\,\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)\Bigg)\Bigg)$$

Problem 82: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\;x\,\sqrt{d+e x}\;dx$$

Optimal (type 4, 550 leaves, 8 steps):

$$-\frac{2\,(2\,a\,d-b\,e)\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\,\sqrt{d+e\,x}}{15\,a\,e}+\frac{2\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\,(d+e\,x)^{3/2}}{5\,e}-$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(a^2\,d^2+b^2\,e^2-a\,e\,(b\,d+3\,c\,e)\right)\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\,\sqrt{d+e\,x}\,\sqrt{-\frac{a\,(c+b\,x+a\,x^2)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,a\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/$$

$$\left(15\,a^2\,e^2\,\sqrt{\frac{a\,(d+e\,x)}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,(c+b\,x+a\,x^2)\right)+$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,(2\,a\,d-b\,e)\,\left(a\,d^2-e\,(b\,d-c\,e)\right)\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\,\sqrt{\frac{a\,(d+e\,x)}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\right.$$

$$\left.\sqrt{-\frac{a\,(c+b\,x+a\,x^2)}{b^2-4\,a\,c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,a\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/$$

$$\left(15\,a^2\,e^2\,\sqrt{d+e\,x}\,(c+b\,x+a\,x^2)\right)$$

Result (type 4, 3390 leaves) :

$$\left(\frac{2\,(a\,d+b\,e)}{15\,a\,e}+\frac{2\,x}{5}\right)\,x\,\sqrt{d+e\,x}\,\sqrt{a+\frac{c+b\,x}{x^2}}\,-$$

$$\frac{1}{15\, a\, e^3 \sqrt{c + b\, x + a\, x^2}} 2\, x \sqrt{a + \frac{c + b\, x}{x^2}} \left(\left(2 \left(a^2 d^2 - a\, b\, d\, e + b^2 e^2 - 3\, a\, c\, e^2\right)\right.\right.$$

$$\left.\left. (d + e\, x)^{3/2} \left(a + \frac{a\, d^2}{(d + e\, x)^2} - \frac{b\, d\, e}{(d + e\, x)^2} + \frac{c\, e^2}{(d + e\, x)^2} - \frac{2\, a\, d}{d + e\, x} + \frac{b\, e}{d + e\, x}\right)\right)\right/$$

$$\left(a \sqrt{\frac{(d + e\, x)^2 \left(a \left(-1 + \frac{d}{d+e\, x}\right)^2 + \frac{e \left(b - \frac{b\, d}{d+e\, x} + \frac{c\, e}{d+e\, x}\right)}{d+e\, x}\right)}{e^2}}\right) - \frac{1}{a \sqrt{\frac{(d+e\, x)^2 \left(a \left(-1+\frac{d}{d+e\, x}\right)^2 + \frac{e \left(b - \frac{b\, d}{d+e\, x} + \frac{c\, e}{d+e\, x}\right)}{d+e\, x}\right)}{e^2}}}$$

$$\left(a\, d^2 - b\, d\, e + c\, e^2\right) (d + e\, x) \sqrt{a + \frac{a\, d^2}{(d + e\, x)^2} - \frac{b\, d\, e}{(d + e\, x)^2} + \frac{c\, e^2}{(d + e\, x)^2} - \frac{2\, a\, d}{d + e\, x} + \frac{b\, e}{d + e\, x}}$$

$$\left(\left(\left(i\, a^2 d^2 \left(2\, a\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}\right) \sqrt{1 - \frac{2 \left(a\, d^2 - b\, d\, e + c\, e^2\right)}{\left(2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}\right) (d + e\, x)}}\right.\right.\right.$$

$$\sqrt{1 - \frac{2 \left(a\, d^2 - b\, d\, e + c\, e^2\right)}{\left(2\, a\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}\right) (d + e\, x)}}$$

$$\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{a\, d^2 - b\, d\, e + c\, e^2}{2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}}}{\sqrt{d + e\, x}}\right], \frac{2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}{2\, a\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}\right] - \right.$$

$$\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{a\, d^2 - b\, d\, e + c\, e^2}{2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}}}{\sqrt{d + e\, x}}\right],\right.$$

$$\left.\left.\left.\frac{2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}{2\, a\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}\right]\right)\right) \Big/ \left(\sqrt{2} \left(a\, d^2 - b\, d\, e + c\, e^2\right)\right.$$

$$\left.\sqrt{-\frac{a\, d^2 - b\, d\, e + c\, e^2}{2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}} \sqrt{a + \frac{a\, d^2 - b\, d\, e + c\, e^2}{(d + e\, x)^2} + \frac{-2\, a\, d + b\, e}{d + e\, x}}\right) -$$

$$\left(\mathbb{i}\, a\, b\, d\, e \left(2\, a\, d - b\, e + \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}\right) \sqrt{1 - \frac{2\left(a\, d^2 - b\, d\, e + c\, e^2\right)}{\left(2\, a\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}\right)(d + e\, x)}}\right.$$

$$\sqrt{1 - \frac{2\left(a\, d^2 - b\, d\, e + c\, e^2\right)}{\left(2\, a\, d - b\, e + \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}\right)(d + e\, x)}}$$

$$\left(\text{EllipticE}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a\, d^2 - b\, d\, e + c\, e^2}{2\, a\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}}}{\sqrt{d + e\, x}}\right], \frac{2\, a\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}{2\, a\, d - b\, e + \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}\right] - \right.$$

$$\text{EllipticF}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a\, d^2 - b\, d\, e + c\, e^2}{2\, a\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}}}{\sqrt{d + e\, x}}\right],\right.$$

$$\left.\left.\frac{2\, a\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}{2\, a\, d - b\, e + \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}\right]\right)\right) \Big/ \left(\sqrt{2}\left(a\, d^2 - b\, d\, e + c\, e^2\right)\right.$$

$$\left.\sqrt{-\frac{a\, d^2 - b\, d\, e + c\, e^2}{2\, a\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}} \sqrt{a + \frac{a\, d^2 - b\, d\, e + c\, e^2}{(d + e\, x)^2} + \frac{-2\, a\, d + b\, e}{d + e\, x}}\right) +$$

$$\left(\mathbb{i}\, b^2\, e^2 \left(2\, a\, d - b\, e + \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}\right) \sqrt{1 - \frac{2\left(a\, d^2 - b\, d\, e + c\, e^2\right)}{\left(2\, a\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}\right)(d + e\, x)}}\right.$$

$$\sqrt{1 - \frac{2\left(a\, d^2 - b\, d\, e + c\, e^2\right)}{\left(2\, a\, d - b\, e + \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}\right)(d + e\, x)}}$$

$$\left(\text{EllipticE}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a\, d^2 - b\, d\, e + c\, e^2}{2\, a\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}}}{\sqrt{d + e\, x}}\right], \frac{2\, a\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}{2\, a\, d - b\, e + \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}\right] - \right.$$

$$\text{EllipticF}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a\, d^2 - b\, d\, e + c\, e^2}{2\, a\, d - b\, e - \sqrt{b^2\, e^2 - 4\, a\, c\, e^2}}}}{\sqrt{d + e\, x}}\right],\right.$$

$$\left.\left.\left.\frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Bigg/\left(\sqrt{2}\,\left(a\,d^2-b\,d\,e+c\,e^2\right)\right.$$

$$\left.\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{a+\frac{a\,d^2-b\,d\,e+c\,e^2}{\left(d+e\,x\right)^2}+\frac{-2\,a\,d+b\,e}{d+e\,x}}\right)-$$

$$\left(3\,\mathrm{i}\,a\,c\,e^2\,\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}\right.$$

$$\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\right.$$

$$\left.\left.\left.\frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Bigg/\left(\sqrt{2}\,\left(a\,d^2-b\,d\,e+c\,e^2\right)\right.$$

$$\left.\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{a+\frac{a\,d^2-b\,d\,e+c\,e^2}{\left(d+e\,x\right)^2}+\frac{-2\,a\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(\mathrm{i}\,\sqrt{2}\,a^2\,d\,\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}\right.$$

$$\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\left(d+e\,x\right)}}$$

$$\text{EllipticF}\left[\text{i ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg/$$

$$\left(\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)-$$

$$\left(\text{i} a b e\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left.\text{EllipticF}\left[\text{i ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\left(\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)\right)$$

Problem 83: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+e x}\,dx$$

Optimal (type 4, 955 leaves, 16 steps):

$$\frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\;x\sqrt{d+e x}\;+$$

$$\left(\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(a\,d+b\,e\right)\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\,x\,\sqrt{d+e\,x}\,\sqrt{-\frac{a\,\left(c+b\,x+a\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,a\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(3\,a\,e\,\sqrt{\frac{a\,\left(d+e\,x\right)}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\left(c+b\,x+a\,x^2\right)\right)-$$

$$\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\,d\,\left(a\,d+b\,e\right)\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\,x\,\sqrt{\frac{a\,\left(d+e\,x\right)}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{a\,\left(c+b\,x+a\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,a\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Big/$$

$$\left(3\,a\,e\,\sqrt{d+e\,x}\,\left(c+b\,x+a\,x^2\right)\right)+\left(4\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(b\,d+c\,e\right)\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\,x\right.$$

$$\sqrt{\frac{a\,\left(d+e\,x\right)}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{a\,\left(c+b\,x+a\,x^2\right)}{b^2-4\,a\,c}}\,\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,a\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],$$

$$-\frac{2\sqrt{b^2-4ac}\ e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\Bigg]\Bigg)\Bigg/\left(3a\sqrt{d+ex}\left(c+bx+ax^2\right)\right)-$$

$$\left(\sqrt{2}\ c\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\ x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\right.$$

$$\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\ \text{EllipticPi}\Big[\frac{2ad-be+\sqrt{b^2-4ac}\ e}{2ad},$$

$$\text{ArcSin}\Big[\frac{\sqrt{2}\sqrt{a}\sqrt{d+ex}}{\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\Big],\frac{b-\sqrt{b^2-4ac}-\frac{2ad}{e}}{b+\sqrt{b^2-4ac}-\frac{2ad}{e}}\Big]\Bigg)\Bigg/\left(\sqrt{a}\left(c+bx+ax^2\right)\right)$$

Result (type 4, 4144 leaves):

$$\frac{2}{3}x\sqrt{d+ex}\sqrt{a+\frac{c+bx}{x^2}}+\frac{1}{3e^2\sqrt{c+bx+ax^2}}$$

$$2x\sqrt{a+\frac{c+bx}{x^2}}\left(\frac{(ad+be)(d+ex)^{3/2}\left(a+\frac{ad^2}{(d+ex)^2}-\frac{bde}{(d+ex)^2}+\frac{ce^2}{(d+ex)^2}-\frac{2ad}{d+ex}+\frac{be}{d+ex}\right)}{a\sqrt{\frac{(d+ex)^2\left(a\left(-1+\frac{d}{d+ex}\right)^2+\frac{e\left(b-\frac{bd}{d+ex}+\frac{ce}{d+ex}\right)}{d+ex}\right)}{e^2}}}-\right.$$

$$\frac{1}{a\sqrt{\frac{(d+ex)^2\left(a\left(-1+\frac{d}{d+ex}\right)^2+\frac{e\left(b-\frac{bd}{d+ex}+\frac{ce}{d+ex}\right)}{d+ex}\right)}{e^2}}}$$

$$(d+ex)\sqrt{a+\frac{ad^2}{(d+ex)^2}-\frac{bde}{(d+ex)^2}+\frac{ce^2}{(d+ex)^2}-\frac{2ad}{d+ex}+\frac{be}{d+ex}}$$

$$\left(\left(i\,a^2d^3\left(2ad-be+\sqrt{b^2e^2-4ace^2}\right)\sqrt{1-\frac{2\left(ad^2-bde+ce^2\right)}{\left(2ad-be-\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}\right.\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\right.$$

$$\left.\left.\left.\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/\left(2\sqrt{2}\left(a d^2-b d e+c e^2\right)\right.$$

$$\left.\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)-$$

$$\left(\mathrm{i}\, b^2 d e^2\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\right.$$

$$\left.\left.\left.\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/\left(2\sqrt{2}\left(a d^2-b d e+c e^2\right)\right.$$

$$\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\Bigg)+$$

$$\Bigg(\mathrm{i}\, a c d e^2 \left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\Bigg(\text{EllipticE}\Big[\mathrm{i}\, \text{ArcSinh}\Big[\frac{\sqrt{2} \sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big], \frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]-$$

$$\text{EllipticF}\Big[\mathrm{i}\, \text{ArcSinh}\Big[\frac{\sqrt{2} \sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big],$$

$$\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]\Bigg)\Bigg) \Bigg/ \Bigg(2 \sqrt{2} \left(a d^2-b d e+c e^2\right)$$

$$\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\Bigg)+$$

$$\Bigg(\mathrm{i}\, b c e^3 \left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\Bigg(\text{EllipticE}\Big[\mathrm{i}\, \text{ArcSinh}\Big[\frac{\sqrt{2} \sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\Big], \frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\Big]-$$

$$\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\right.$$

$$\left.\left.\left.\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/\left(2\sqrt{2}\,\left(a d^2-b d e+c e^2\right)\right.$$

$$\left.\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)+$$

$$\left(\mathbb{i}\,a^2 d^2\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)-$$

$$\left(\mathbb{i}\,a\,b\,d\,e\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)+$$

$$\left(\mathrm{i}\, a c e^2\,\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)-$$

$$\left(3\,\mathrm{i}\, a c e^2\,\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticPi}\left[\frac{d\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)}{2\left(a d^2-b d e+c e^2\right)},\right.$$

$$\left.\left.\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left.\left(\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\,\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)\right)\right)$$

Problem 84: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\ \sqrt{d+e x}}{x}\, dx$$

Optimal (type 4, 929 leaves, 16 steps):

$$-\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\ \sqrt{d+e x}+$$

$$\left(3\sqrt{b^2-4 a c}\ \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\ x\sqrt{d+e x}\ \sqrt{-\frac{a\left(c+b x+a x^2\right)}{b^2-4 a c}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 a x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],\right.\right.$$

$$\left.\left.-\frac{2\sqrt{b^2-4 a c}\ e}{2 a d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Big/\left(\sqrt{2}\ \sqrt{\frac{a\left(d+e x\right)}{2 a d-\left(b+\sqrt{b^2-4 a c}\right) e}}\ \left(c+b x+a x^2\right)\right)-$$

$$\left(3\sqrt{2}\ \sqrt{b^2-4 a c}\ d\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\ x\sqrt{\frac{a\left(d+e x\right)}{2 a d-\left(b+\sqrt{b^2-4 a c}\right) e}}\ \sqrt{-\frac{a\left(c+b x+a x^2\right)}{b^2-4 a c}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 a x}{\sqrt{b^2-4 a c}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4 a c}\ e}{2 a d-\left(b+\sqrt{b^2-4 a c}\right) e}\right]\right)\Big/$$

$$\left(\sqrt{d+e\,x}\,\left(c+b\,x+a\,x^2\right)\right)+\left(2\,\sqrt{2}\,\sqrt{b^2-4\,a\,c}\,\left(a\,d+b\,e\right)\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right.$$

$$\sqrt{\frac{a\,\left(d+e\,x\right)}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{a\,\left(c+b\,x+a\,x^2\right)}{b^2-4\,a\,c}}\,\text{EllipticF}\Big[$$

$$\left.\text{ArcSin}\Big[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,a\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\Big],\,-\frac{2\,\sqrt{b^2-4\,a\,c}\,e}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\Big]\right)\Big/\left(a\,\sqrt{d+e\,x}\,\left(c+b\,x+a\,x^2\right)\right)-$$

$$\left(\left(b\,d+c\,e\right)\,\sqrt{2\,a\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)\,e}\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\,\sqrt{1-\frac{2\,a\,\left(d+e\,x\right)}{2\,a\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)\,e}}\right.$$

$$\sqrt{1-\frac{2\,a\,\left(d+e\,x\right)}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\text{EllipticPi}\Big[\frac{2\,a\,d-b\,e+\sqrt{b^2-4\,a\,c}\,e}{2\,a\,d},$$

$$\left.\text{ArcSin}\Big[\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{d+e\,x}}{\sqrt{2\,a\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)\,e}}\Big],\,\frac{b-\sqrt{b^2-4\,a\,c}-\frac{2\,a\,d}{e}}{b+\sqrt{b^2-4\,a\,c}-\frac{2\,a\,d}{e}}\Big]\right)\Big/\left(\sqrt{2}\,\sqrt{a}\,d\,\left(c+b\,x+a\,x^2\right)\right)$$

Result (type 4, 4893 leaves) :

$$-\sqrt{d+e\,x}\,\sqrt{a+\frac{c+b\,x}{x^2}}+$$

$$\frac{1}{e\,\sqrt{c+b\,x+a\,x^2}}x\,\sqrt{a+\frac{c+b\,x}{x^2}}\,\left(\frac{3\,\left(d+e\,x\right)^{3/2}\,\left(a+\frac{a\,d^2}{(d+e\,x)^2}-\frac{b\,d\,e}{(d+e\,x)^2}+\frac{c\,e^2}{(d+e\,x)^2}-\frac{2\,a\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}\right)}{\sqrt{\frac{(d+e\,x)^2\,\left(a\,\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\,\left(b-\frac{b\,d}{d+e\,x}+\frac{c\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}}-\right.$$

$$\left(3\, i\, a\, d^2 \left(2\, a\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}\right) (d + e\, x)\right.$$

$$\sqrt{a + \frac{a\, d^2}{(d + e\, x)^2} - \frac{b\, d\, e}{(d + e\, x)^2} + \frac{c\, e^2}{(d + e\, x)^2} - \frac{2\, a\, d}{d + e\, x} + \frac{b\, e}{d + e\, x}}$$

$$\sqrt{1 - \frac{2\left(a\, d^2 - b\, d\, e + c\, e^2\right)}{\left(2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}\right)(d + e\, x)}}$$

$$\sqrt{1 - \frac{2\left(a\, d^2 - b\, d\, e + c\, e^2\right)}{\left(2\, a\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}\right)(d + e\, x)}}$$

$$\left(\text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a\, d^2 - b\, d\, e + c\, e^2}{2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}}}{\sqrt{d + e\, x}}\right], \frac{2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}{2\, a\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}\right] - \right.$$

$$\left.\left.\text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a\, d^2 - b\, d\, e + c\, e^2}{2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}}}{\sqrt{d + e\, x}}\right], \frac{2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}{2\, a\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}\right]\right)\right) \Big/$$

$$\left(2\sqrt{2}\left(a\, d^2 - b\, d\, e + c\, e^2\right)\sqrt{-\frac{a\, d^2 - b\, d\, e + c\, e^2}{2\, a\, d - b\, e - \sqrt{b^2 e^2 - 4\, a\, c\, e^2}}}\right.$$

$$\left.\sqrt{a + \frac{a\, d^2 - b\, d\, e + c\, e^2}{(d + e\, x)^2} + \frac{-2\, a\, d + b\, e}{d + e\, x}}\,\sqrt{\frac{(d + e\, x)^2\left(a\left(-1 + \frac{d}{d+e\, x}\right)^2 + \frac{e\left(b - \frac{b\, d}{d+e\, x} + \frac{c\, e}{d+e\, x}\right)}{d+e\, x}\right)}{e^2}}\right) +$$

$$\left(3\, i\, b\, d\, e\left(2\, a\, d - b\, e + \sqrt{b^2 e^2 - 4\, a\, c\, e^2}\right)(d + e\, x)\right.$$

$$\sqrt{a+\frac{a d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{c e^2}{(d+e x)^2}-\frac{2 a d}{d+e x}+\frac{b e}{d+e x}}$$

$$\sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right) \Big/$$

$$\left(2 \sqrt{2} \left(a d^2-b d e+c e^2\right) \sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\right.$$

$$\left.\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}} \sqrt{\frac{(d+e x)^2 \left(a \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{c e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right) -$$

$$\left(3 i c e^2 \left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)\right.$$

$$\sqrt{a+\frac{a d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{c e^2}{(d+e x)^2}-\frac{2 a d}{d+e x}+\frac{b e}{d+e x}}$$

$$\sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(a d^2-b d e+c e^2\right)\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\right.$$

$$\left.\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\sqrt{\frac{(d+e x)^2\left(a\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{c e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-$$

$$\left(\mathrm{i}\,a d(d+e x)\sqrt{a+\frac{a d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{c e^2}{(d+e x)^2}-\frac{2 a d}{d+e x}+\frac{b e}{d+e x}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right.$$

$$\left.\sqrt{\frac{(d+e x)^2\left(a\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{c e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+$$

$$\left(\mathrm{i}\, b e (d+e x)\sqrt{a+\frac{a d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{c e^2}{(d+e x)^2}-\frac{2 a d}{d+e x}+\frac{b e}{d+e x}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right.$$

$$\left.\sqrt{\frac{(d+e x)^2\left(a\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{c e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)-$$

$$\left(\mathrm{i}\, c e^2 (d+e x)\sqrt{a+\frac{a d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{c e^2}{(d+e x)^2}-\frac{2 a d}{d+e x}+\frac{b e}{d+e x}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,d\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right.$$

$$\left.\sqrt{\frac{(d+e x)^2\left(a\left(-1+\frac{d}{d+e x}\right)^2+\frac{e\left(b-\frac{b d}{d+e x}+\frac{c e}{d+e x}\right)}{d+e x}\right)}{e^2}}\right)+$$

$$\left(\mathrm{i}\,b e(d+e x)\sqrt{a+\frac{a d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{c e^2}{(d+e x)^2}-\frac{2 a d}{d+e x}+\frac{b e}{d+e x}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticPi}\left[\frac{d\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)}{2\left(a d^2-b d e+c e^2\right)},\right.$$

$$\left.\left.\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{a+\frac{a\,d^2-b\,d\,e+c\,e^2}{\left(d+e\,x\right)^2}+\frac{-2\,a\,d+b\,e}{d+e\,x}}\right.$$

$$\left.\sqrt{\frac{\left(d+e\,x\right)^2\left(a\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{c\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)+$$

$$\left(\mathrm{i}\,c\,e^2\left(d+e\,x\right)\sqrt{a+\frac{a\,d^2}{\left(d+e\,x\right)^2}-\frac{b\,d\,e}{\left(d+e\,x\right)^2}+\frac{c\,e^2}{\left(d+e\,x\right)^2}-\frac{2\,a\,d}{d+e\,x}+\frac{b\,e}{d+e\,x}}\right.$$

$$\sqrt{1-\frac{2\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\left(d+e\,x\right)}}$$

$$\sqrt{1-\frac{2\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\left(d+e\,x\right)}}$$

$$\text{EllipticPi}\left[\frac{d\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)}{2\left(a\,d^2-b\,d\,e+c\,e^2\right)},\right.$$

$$\left.\left.\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,d\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\,\sqrt{a+\frac{a\,d^2-b\,d\,e+c\,e^2}{\left(d+e\,x\right)^2}+\frac{-2\,a\,d+b\,e}{d+e\,x}}\right.$$

$$\left.\left.\sqrt{\frac{\left(d+e\,x\right)^2\left(a\left(-1+\frac{d}{d+e\,x}\right)^2+\frac{e\left(b-\frac{b\,d}{d+e\,x}+\frac{c\,e}{d+e\,x}\right)}{d+e\,x}\right)}{e^2}}\right)\right)$$

Problem 85: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+e\,x}}{x^2}\,dx$$

Optimal (type 4, 1287 leaves, 24 steps):

$$-\frac{(b\,d+c\,e)\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+e\,x}}{4\,c\,d}-\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+e\,x}}{2\,x}+$$

$$\left(\sqrt{b^2-4\,a\,c}\,\,(b\,d+c\,e)\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\,x\,\sqrt{d+e\,x}\,\sqrt{-\frac{a\,\left(c+b\,x+a\,x^2\right)}{b^2-4\,a\,c}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}\,+2\,a\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/$$

$$\left(4\,\sqrt{2}\,\,c\,d\,\sqrt{\frac{a\,(d+e\,x)}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\,\left(c+b\,x+a\,x^2\right)\right)+$$

$$\left(3\,\sqrt{b^2-4\,a\,c}\,\,e\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\,x\,\sqrt{\frac{a\,(d+e\,x)}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}}\,\sqrt{-\frac{a\,\left(c+b\,x+a\,x^2\right)}{b^2-4\,a\,c}}\,\,\text{EllipticF}\left[\right.\right.$$

$$\left.\left.\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}\,+2\,a\,x}{\sqrt{b^2-4\,a\,c}}}}{\sqrt{2}}\right],-\frac{2\,\sqrt{b^2-4\,a\,c}\,\,e}{2\,a\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)\,e}\right]\right)\Bigg/\left(\sqrt{2}\,\,\sqrt{d+e\,x}\,\,\left(c+b\,x+a\,x^2\right)\right)-$$

$$\left(\sqrt{b^2-4ac}\,(b d+c e)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{\frac{a(d+e x)}{2 a d-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a\left(c+b x+a x^2\right)}{b^2-4ac}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right],-\frac{2\sqrt{b^2-4ac}\,e}{2 a d-\left(b+\sqrt{b^2-4ac}\right)e}\right]\right)\Big/$$

$$\left(2\sqrt{2}\,c\sqrt{d+e x}\left(c+b x+a x^2\right)\right)-$$

$$\left((a d+b e)\sqrt{2 a d-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{1-\frac{2a(d+e x)}{2 a d-\left(b-\sqrt{b^2-4ac}\right)e}}\right.$$

$$\sqrt{1-\frac{2a(d+e x)}{2 a d-\left(b+\sqrt{b^2-4ac}\right)e}}\,\text{EllipticPi}\left[\frac{2 a d-b e+\sqrt{b^2-4ac}\,e}{2 a d},\right.$$

$$\left.\left.\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{a}\sqrt{d+e x}}{\sqrt{2 a d-\left(b-\sqrt{b^2-4ac}\right)e}}\right],\frac{b-\sqrt{b^2-4ac}-\frac{2ad}{e}}{b+\sqrt{b^2-4ac}-\frac{2ad}{e}}\right]\right)\Big/\left(\sqrt{2}\sqrt{a}\,d\left(c+b x+a x^2\right)\right)+$$

$$\left((b d+c e)^2\sqrt{2 a d-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{1-\frac{2a(d+e x)}{2 a d-\left(b-\sqrt{b^2-4ac}\right)e}}\right.$$

$$\sqrt{1-\frac{2a(d+e x)}{2 a d-\left(b+\sqrt{b^2-4ac}\right)e}}\,\text{EllipticPi}\left[\frac{2 a d-b e+\sqrt{b^2-4ac}\,e}{2 a d},\text{ArcSin}\left[\right.\right.$$

$$\left.\left.\left.\frac{\sqrt{2}\sqrt{a}\sqrt{d+e x}}{\sqrt{2 a d-\left(b-\sqrt{b^2-4ac}\right)e}}\right],\frac{b-\sqrt{b^2-4ac}-\frac{2ad}{e}}{b+\sqrt{b^2-4ac}-\frac{2ad}{e}}\right]\right)\Big/\left(4\sqrt{2}\sqrt{a}\,c\,d^2\left(c+b x+a x^2\right)\right)$$

Result (type 4, 6206 leaves) :

$$\left(-\frac{1}{2 x^2}+\frac{-b d-c e}{4 c d x}\right) x \sqrt{d+e x} \sqrt{a+\frac{c+b x}{x^2}}+\frac{1}{4 c d e \sqrt{c+b x+a x^2}}$$

$$x \sqrt{a+\frac{c+b x}{x^2}} \left(\frac{(b d+c e) (d+e x)^{3/2} \left(a+\frac{a d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{c e^2}{(d+e x)^2}-\frac{2 a d}{d+e x}+\frac{b e}{d+e x}\right)}{\sqrt{\frac{(d+e x)^2 \left(a \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{c e}{d+e x}\right)}{d+e x}\right)}{e^2}}}-\right.$$

$$\frac{1}{\sqrt{\frac{(d+e x)^2 \left(a \left(-1+\frac{d}{d+e x}\right)^2+\frac{e \left(b-\frac{b d}{d+e x}+\frac{c e}{d+e x}\right)}{d+e x}\right)}{e^2}}}$$

$$d (d+e x) \sqrt{a+\frac{a d^2}{(d+e x)^2}-\frac{b d e}{(d+e x)^2}+\frac{c e^2}{(d+e x)^2}-\frac{2 a d}{d+e x}+\frac{b e}{d+e x}}$$

$$\left(\left(\mathbb{i}\, a b d^2 \left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) \sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}\right.\right.$$

$$\sqrt{1-\frac{2 \left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right) (d+e x)}}$$

$$\left(\text{EllipticE}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right], \frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]-\right.$$

$$\text{EllipticF}\left[\mathbb{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\right.$$

$$\left.\left.\left.\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\right) \Big/ \left(2 \sqrt{2} \left(a d^2-b d e+c e^2\right)\right.$$

$$\left.\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}} \sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)-$$

$$\left(\mathrm{i}\, b^2 d e \left(2 a d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(a d^2 - b d e + c e^2\right)}{\left(2 a d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(a d^2 - b d e + c e^2\right)}{\left(2 a d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}$$

$$\left(\text{EllipticE}\left[\mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{a d^2 - b d e + c e^2}{2 a d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 a d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 a d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{a d^2 - b d e + c e^2}{2 a d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right],\right.$$

$$\left.\left.\left.\frac{2 a d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 a d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right]\right)\right) \Big/ \left(2 \sqrt{2} \left(a d^2 - b d e + c e^2\right)\right.$$

$$\left.\sqrt{-\frac{a d^2 - b d e + c e^2}{2 a d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}} \sqrt{a + \frac{a d^2 - b d e + c e^2}{(d + e x)^2} + \frac{-2 a d + b e}{d + e x}}\right) +$$

$$\left(\mathrm{i}\, a c d e \left(2 a d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) \sqrt{1 - \frac{2 \left(a d^2 - b d e + c e^2\right)}{\left(2 a d - b e - \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}\right.$$

$$\sqrt{1 - \frac{2 \left(a d^2 - b d e + c e^2\right)}{\left(2 a d - b e + \sqrt{b^2 e^2 - 4 a c e^2}\right) (d + e x)}}$$

$$\left(\text{EllipticE}\left[\mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{a d^2 - b d e + c e^2}{2 a d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right], \frac{2 a d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}{2 a d - b e + \sqrt{b^2 e^2 - 4 a c e^2}}\right] - \right.$$

$$\text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{a d^2 - b d e + c e^2}{2 a d - b e - \sqrt{b^2 e^2 - 4 a c e^2}}}}{\sqrt{d + e x}}\right],\right.$$

$$\left.\left.\left.\frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Bigg/\left(2\,\sqrt{2}\,\left(a\,d^2-b\,d\,e+c\,e^2\right)\right.$$

$$\left.\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{a+\frac{a\,d^2-b\,d\,e+c\,e^2}{(d+e\,x)^2}+\frac{-2\,a\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(i\,c^2\,e^3\,\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\,\frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]-\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}}{\sqrt{d+e\,x}}\right],\right.$$

$$\left.\left.\left.\frac{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}{2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}}\right]\right)\right)\Bigg/\left(2\,\sqrt{2}\,d\,\left(a\,d^2-b\,d\,e+c\,e^2\right)\right.$$

$$\left.\sqrt{-\frac{a\,d^2-b\,d\,e+c\,e^2}{2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}}}\;\sqrt{a+\frac{a\,d^2-b\,d\,e+c\,e^2}{(d+e\,x)^2}+\frac{-2\,a\,d+b\,e}{d+e\,x}}\right)+$$

$$\left(i\,a\,b\,d\,\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e-\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}\right.$$

$$\sqrt{1-\frac{2\,\left(a\,d^2-b\,d\,e+c\,e^2\right)}{\left(2\,a\,d-b\,e+\sqrt{b^2\,e^2-4\,a\,c\,e^2}\right)\,(d+e\,x)}}$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)-$$

$$\Bigg(\mathrm{i}\,b^2 e\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)-$$

$$\Bigg(\mathrm{i}\,a c e\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)+$$

$$\left(\mathbb{i}\sqrt{2}\, b\, c\, e^2 \sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(d\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)-$$

$$\left(\mathbb{i}\, c^2 e^3 \sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\left.\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Big/$$

$$\left(\sqrt{2}\, d^2\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\sqrt{a+\frac{a d^2-b d e+c e^2}{(d+e x)^2}+\frac{-2 a d+b e}{d+e x}}\right)+$$

$$\left(\mathbb{i}\, b^2 e \sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)(d+e x)}}$$

$$\text{EllipticPi}\Big[\frac{d\left(2ad-be-\sqrt{b^2e^2-4ace^2}\right)}{2\left(ad^2-bde+ce^2\right)},\ \text{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\sqrt{-\frac{ad^2-bde+ce^2}{2ad-be-\sqrt{b^2e^2-4ace^2}}}}{\sqrt{d+ex}}\Big],\ \frac{2ad-be-\sqrt{b^2e^2-4ace^2}}{2ad-be+\sqrt{b^2e^2-4ace^2}}\Big]\Bigg)\Bigg/$$

$$\left(\sqrt{2}\sqrt{-\frac{ad^2-bde+ce^2}{2ad-be-\sqrt{b^2e^2-4ace^2}}}\sqrt{a+\frac{ad^2-bde+ce^2}{(d+ex)^2}+\frac{-2ad+be}{d+ex}}\right)-$$

$$\Bigg(2\,\text{i}\sqrt{2}\,ace\sqrt{1-\frac{2\left(ad^2-bde+ce^2\right)}{\left(2ad-be-\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}$$

$$\sqrt{1-\frac{2\left(ad^2-bde+ce^2\right)}{\left(2ad-be+\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}$$

$$\text{EllipticPi}\Big[\frac{d\left(2ad-be-\sqrt{b^2e^2-4ace^2}\right)}{2\left(ad^2-bde+ce^2\right)},\ \text{i}\,\text{ArcSinh}\Big[\frac{\sqrt{2}\sqrt{-\frac{ad^2-bde+ce^2}{2ad-be-\sqrt{b^2e^2-4ace^2}}}}{\sqrt{d+ex}}\Big],\ \frac{2ad-be-\sqrt{b^2e^2-4ace^2}}{2ad-be+\sqrt{b^2e^2-4ace^2}}\Big]\Bigg)\Bigg/$$

$$\left(\sqrt{-\frac{ad^2-bde+ce^2}{2ad-be-\sqrt{b^2e^2-4ace^2}}}\sqrt{a+\frac{ad^2-bde+ce^2}{(d+ex)^2}+\frac{-2ad+be}{d+ex}}\right)-$$

$$\Bigg(\text{i}\sqrt{2}\,bce^2\sqrt{1-\frac{2\left(ad^2-bde+ce^2\right)}{\left(2ad-be-\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}$$

$$\sqrt{1-\frac{2\left(ad^2-bde+ce^2\right)}{\left(2ad-be+\sqrt{b^2e^2-4ace^2}\right)(d+ex)}}$$

$$\text{EllipticPi}\Big[\frac{d\left(2ad-be-\sqrt{b^2e^2-4ace^2}\right)}{2\left(ad^2-bde+ce^2\right)},$$

$$\left.\left.\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left(d\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{a+\frac{a d^2-b d e+c e^2}{\left(d+e x\right)^2}+\frac{-2 a d+b e}{d+e x}}\right)+$$

$$\left(\mathbb{i}\,c^2\,e^3\,\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)\left(d+e x\right)}}\right.$$

$$\sqrt{1-\frac{2\left(a d^2-b d e+c e^2\right)}{\left(2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}\right)\left(d+e x\right)}}$$

$$\text{EllipticPi}\left[\frac{d\left(2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}\right)}{2\left(a d^2-b d e+c e^2\right)},\right.$$

$$\left.\left.\mathbb{i}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}}{\sqrt{d+e x}}\right],\ \frac{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}{2 a d-b e+\sqrt{b^2 e^2-4 a c e^2}}\right]\right)\Bigg/$$

$$\left.\left(\sqrt{2}\,d^2\,\sqrt{-\frac{a d^2-b d e+c e^2}{2 a d-b e-\sqrt{b^2 e^2-4 a c e^2}}}\ \sqrt{a+\frac{a d^2-b d e+c e^2}{\left(d+e x\right)^2}+\frac{-2 a d+b e}{d+e x}}\right)\right)\right)$$

Problem 90: Unable to integrate problem.

$$\int\frac{\left(f x\right)^m\left(a+c x^{2 n}\right)^p}{d+e x^n}\,dx$$

Optimal (type 6, 194 leaves, 6 steps):

$$\frac{1}{d\left(1+m\right)}$$

$$x\left(f x\right)^m\left(a+c x^{2 n}\right)^p\left(1+\frac{c x^{2 n}}{a}\right)^{-p}\text{AppellF1}\left[\frac{1+m}{2 n},-p,1,1+\frac{1+m}{2 n},-\frac{c x^{2 n}}{a},\frac{e^2 x^{2 n}}{d^2}\right]-\frac{1}{d^2\left(1+m+n\right)}$$

$$e\,x^{1+n}\left(f x\right)^m\left(a+c x^{2 n}\right)^p\left(1+\frac{c x^{2 n}}{a}\right)^{-p}\text{AppellF1}\left[\frac{1+m+n}{2 n},-p,1,\frac{1+m+3 n}{2 n},-\frac{c x^{2 n}}{a},\frac{e^2 x^{2 n}}{d^2}\right]$$

Result (type 8, 28 leaves):

$$\int \frac{(f x)^m \left(a + c x^{2 n}\right)^p}{d + e x^n} \, dx$$

Problem 91: Unable to integrate problem.

$$\int \frac{(f x)^m \left(a + c x^{2 n}\right)^p}{\left(d + e x^n\right)^2} \, dx$$

Optimal (type 6, 302 leaves, 8 steps):

$$\frac{1}{d^2 (1+m)} x (f x)^m \left(a + c x^{2 n}\right)^p \left(1 + \frac{c x^{2 n}}{a}\right)^{-p} \text{AppellF1}\left[\frac{1+m}{2 n}, -p, 2, 1 + \frac{1+m}{2 n}, -\frac{c x^{2 n}}{a}, \frac{e^2 x^{2 n}}{d^2}\right] - \frac{1}{d^3 (1+m+n)} 2 e x^{1+n} (f x)^m \left(a + c x^{2 n}\right)^p \left(1 + \frac{c x^{2 n}}{a}\right)^{-p} \text{AppellF1}\left[\frac{1+m+n}{2 n}, -p, 2, \frac{1+m+3 n}{2 n}, -\frac{c x^{2 n}}{a}, \frac{e^2 x^{2 n}}{d^2}\right] + \frac{1}{d^4 (1+m+2 n)} e^2 x^{1+2 n} (f x)^m \left(a + c x^{2 n}\right)^p \left(1 + \frac{c x^{2 n}}{a}\right)^{-p} \text{AppellF1}\left[\frac{1+m+2 n}{2 n}, -p, 2, \frac{1+m+4 n}{2 n}, -\frac{c x^{2 n}}{a}, \frac{e^2 x^{2 n}}{d^2}\right]$$

Result (type 8, 28 leaves):

$$\int \frac{(f x)^m \left(a + c x^{2 n}\right)^p}{\left(d + e x^n\right)^2} \, dx$$

Problem 92: Unable to integrate problem.

$$\int \frac{(f x)^m \left(a + c x^{2 n}\right)^p}{\left(d + e x^n\right)^3} \, dx$$

Optimal (type 6, 412 leaves, 10 steps):

$$\frac{1}{d^3 (1+m)} x (f x)^m \left(a + c x^{2 n}\right)^p \left(1 + \frac{c x^{2 n}}{a}\right)^{-p} \text{AppellF1}\left[\frac{1+m}{2 n}, -p, 3, 1 + \frac{1+m}{2 n}, -\frac{c x^{2 n}}{a}, \frac{e^2 x^{2 n}}{d^2}\right] - \frac{1}{d^4 (1+m+n)} 3 e x^{1+n} (f x)^m \left(a + c x^{2 n}\right)^p \left(1 + \frac{c x^{2 n}}{a}\right)^{-p} \text{AppellF1}\left[\frac{1+m+n}{2 n}, -p, 3, \frac{1+m+3 n}{2 n}, -\frac{c x^{2 n}}{a}, \frac{e^2 x^{2 n}}{d^2}\right] + \frac{1}{d^5 (1+m+2 n)} 3 e^2 x^{1+2 n} (f x)^m \left(a + c x^{2 n}\right)^p \left(1 + \frac{c x^{2 n}}{a}\right)^{-p} \text{AppellF1}\left[\frac{1+m+2 n}{2 n}, -p, 3, \frac{1+m+4 n}{2 n}, -\frac{c x^{2 n}}{a}, \frac{e^2 x^{2 n}}{d^2}\right] - \frac{1}{d^6 (1+m+3 n)} e^3 x^{1+3 n} (f x)^m \left(a + c x^{2 n}\right)^p \left(1 + \frac{c x^{2 n}}{a}\right)^{-p} \text{AppellF1}\left[\frac{1+m+3 n}{2 n}, -p, 3, \frac{1+m+5 n}{2 n}, -\frac{c x^{2 n}}{a}, \frac{e^2 x^{2 n}}{d^2}\right]$$

Result (type 8, 28 leaves):

$$\int \frac{(f x)^m \left(a+c x^{2 n}\right)^p}{\left(d+e x^n\right)^3} \, dx$$

Problem 93: Result more than twice size of optimal antiderivative.

$$\int (b+2 c x) \left(a+b x+c x^2\right)^{13} \, dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{1}{14} \left(a+b x+c x^2\right)^{14}$$

Result (type 1, 201 leaves):

$$\frac{1}{14} x (b+c x) \left(14 a^{13}+91 a^{12} x (b+c x)+364 a^{11} x^2 (b+c x)^2+1001 a^{10} x^3 (b+c x)^3+2002 a^9 x^4 (b+c x)^4+3003 a^8 x^5 (b+c x)^5+3432 a^7 x^6 (b+c x)^6+3003 a^6 x^7 (b+c x)^7+2002 a^5 x^8 (b+c x)^8+1001 a^4 x^9 (b+c x)^9+364 a^3 x^{10} (b+c x)^{10}+91 a^2 x^{11} (b+c x)^{11}+14 a x^{12} (b+c x)^{12}+x^{13} (b+c x)^{13}\right)$$

Problem 94: Result more than twice size of optimal antiderivative.

$$\int x \left(b+2 c x^2\right) \left(a+b x^2+c x^4\right)^{13} \, dx$$

Optimal (type 1, 18 leaves, 2 steps):

$$\frac{1}{28} \left(a+b x^2+c x^4\right)^{14}$$

Result (type 1, 233 leaves):

$$\frac{1}{28} x^2 \left(b+c x^2\right) \left(14 a^{13}+91 a^{12} x^2 \left(b+c x^2\right)+364 a^{11} x^4 \left(b+c x^2\right)^2+1001 a^{10} x^6 \left(b+c x^2\right)^3+2002 a^9 x^8 \left(b+c x^2\right)^4+3003 a^8 x^{10} \left(b+c x^2\right)^5+3432 a^7 x^{12} \left(b+c x^2\right)^6+3003 a^6 x^{14} \left(b+c x^2\right)^7+2002 a^5 x^{16} \left(b+c x^2\right)^8+1001 a^4 x^{18} \left(b+c x^2\right)^9+364 a^3 x^{20} \left(b+c x^2\right)^{10}+91 a^2 x^{22} \left(b+c x^2\right)^{11}+14 a x^{24} \left(b+c x^2\right)^{12}+x^{26} \left(b+c x^2\right)^{13}\right)$$

Problem 95: Result more than twice size of optimal antiderivative.

$$\int x^2 \left(b+2 c x^3\right) \left(a+b x^3+c x^6\right)^{13} \, dx$$

Optimal (type 1, 18 leaves, 2 steps):

$$\frac{1}{42} \left(a+b x^3+c x^6\right)^{14}$$

Result (type 1, 233 leaves):

$$\frac{1}{42} x^3 \left(b+c x^3\right) \left(14 a^{13}+91 a^{12} x^3 \left(b+c x^3\right)+364 a^{11} x^6 \left(b+c x^3\right)^2+1001 a^{10} x^9 \left(b+c x^3\right)^3+2002 a^9 x^{12} \left(b+c x^3\right)^4+3003 a^8 x^{15} \left(b+c x^3\right)^5+3432 a^7 x^{18} \left(b+c x^3\right)^6+3003 a^6 x^{21} \left(b+c x^3\right)^7+2002 a^5 x^{24} \left(b+c x^3\right)^8+1001 a^4 x^{27} \left(b+c x^3\right)^9+364 a^3 x^{30} \left(b+c x^3\right)^{10}+91 a^2 x^{33} \left(b+c x^3\right)^{11}+14 a x^{36} \left(b+c x^3\right)^{12}+x^{39} \left(b+c x^3\right)^{13}\right)$$

Problem 96: Result more than twice size of optimal antiderivative.

$$\int x^{-1+n} \left(b+2 c x^n\right) \left(a+b x^n+c x^{2 n}\right)^{13} \, dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$\frac{\left(a+b x^n+c x^{2 n}\right)^{14}}{14 n}$$

Result (type 3, 260 leaves):

$$\frac{1}{14 n} x^n \left(b+c x^n\right) \left(14 a^{13}+91 a^{12} x^n \left(b+c x^n\right)+364 a^{11} x^{2 n} \left(b+c x^n\right)^2+1001 a^{10} x^{3 n} \left(b+c x^n\right)^3+2002 a^9 x^{4 n} \left(b+c x^n\right)^4+3003 a^8 x^{5 n} \left(b+c x^n\right)^5+3432 a^7 x^{6 n} \left(b+c x^n\right)^6+3003 a^6 x^{7 n} \left(b+c x^n\right)^7+2002 a^5 x^{8 n} \left(b+c x^n\right)^8+1001 a^4 x^{9 n} \left(b+c x^n\right)^9+364 a^3 x^{10 n} \left(b+c x^n\right)^{10}+91 a^2 x^{11 n} \left(b+c x^n\right)^{11}+14 a x^{12 n} \left(b+c x^n\right)^{12}+x^{13 n} \left(b+c x^n\right)^{13}\right)$$

Problem 97: Result more than twice size of optimal antiderivative.

$$\int \left(b+2 c x\right) \left(-a+b x+c x^2\right)^{13} \, dx$$

Optimal (type 1, 18 leaves, 1 step):

$$\frac{1}{14} \left(a-b x-c x^2\right)^{14}$$

Result (type 1, 201 leaves):

$$\frac{1}{14} x \left(b+c x\right) \left(-14 a^{13}+91 a^{12} x \left(b+c x\right)-364 a^{11} x^2 \left(b+c x\right)^2+1001 a^{10} x^3 \left(b+c x\right)^3-2002 a^9 x^4 \left(b+c x\right)^4+3003 a^8 x^5 \left(b+c x\right)^5-3432 a^7 x^6 \left(b+c x\right)^6+3003 a^6 x^7 \left(b+c x\right)^7-2002 a^5 x^8 \left(b+c x\right)^8+1001 a^4 x^9 \left(b+c x\right)^9-364 a^3 x^{10} \left(b+c x\right)^{10}+91 a^2 x^{11} \left(b+c x\right)^{11}-14 a x^{12} \left(b+c x\right)^{12}+x^{13} \left(b+c x\right)^{13}\right)$$

Problem 98: Result more than twice size of optimal antiderivative.

$$\int x \left(b+2 c x^2\right) \left(-a+b x^2+c x^4\right)^{13} \, dx$$

Optimal (type 1, 20 leaves, 2 steps):

$$\frac{1}{28} \left(a-b x^2-c x^4\right)^{14}$$

Result (type 1, 233 leaves):

$$\frac{1}{28} x^2 \left(b+c x^2\right) \left(-14 a^{13}+91 a^{12} x^2 \left(b+c x^2\right)-364 a^{11} x^4 \left(b+c x^2\right)^2+1001 a^{10} x^6 \left(b+c x^2\right)^3-2002 a^9 x^8 \left(b+c x^2\right)^4+3003 a^8 x^{10} \left(b+c x^2\right)^5-3432 a^7 x^{12} \left(b+c x^2\right)^6+3003 a^6 x^{14} \left(b+c x^2\right)^7-2002 a^5 x^{16} \left(b+c x^2\right)^8+1001 a^4 x^{18} \left(b+c x^2\right)^9-364 a^3 x^{20} \left(b+c x^2\right)^{10}+91 a^2 x^{22} \left(b+c x^2\right)^{11}-14 a x^{24} \left(b+c x^2\right)^{12}+x^{26} \left(b+c x^2\right)^{13}\right)$$

Problem 99: Result more than twice size of optimal antiderivative.

$$\int x^2 \left(b+2 c x^3\right) \left(-a+b x^3+c x^6\right)^{13} \, dx$$

Optimal (type 1, 20 leaves, 2 steps):

$$\frac{1}{42} \left(a-b x^3-c x^6\right)^{14}$$

Result (type 1, 233 leaves):

$$\frac{1}{42} x^3 \left(b+c x^3\right) \left(-14 a^{13}+91 a^{12} x^3 \left(b+c x^3\right)-364 a^{11} x^6 \left(b+c x^3\right)^2+1001 a^{10} x^9 \left(b+c x^3\right)^3-2002 a^9 x^{12} \left(b+c x^3\right)^4+3003 a^8 x^{15} \left(b+c x^3\right)^5-3432 a^7 x^{18} \left(b+c x^3\right)^6+3003 a^6 x^{21} \left(b+c x^3\right)^7-2002 a^5 x^{24} \left(b+c x^3\right)^8+1001 a^4 x^{27} \left(b+c x^3\right)^9-364 a^3 x^{30} \left(b+c x^3\right)^{10}+91 a^2 x^{33} \left(b+c x^3\right)^{11}-14 a x^{36} \left(b+c x^3\right)^{12}+x^{39} \left(b+c x^3\right)^{13}\right)$$

Problem 100: Result more than twice size of optimal antiderivative.

$$\int x^{-1+n} \left(b+2 c x^n\right) \left(-a+b x^n+c x^{2 n}\right)^{13} \, dx$$

Optimal (type 3, 25 leaves, 2 steps):

$$\frac{\left(a-b x^n-c x^{2 n}\right)^{14}}{14 n}$$

Result (type 3, 260 leaves):

$$\frac{1}{14 n} x^n \left(b+c x^n\right) \left(-14 a^{13}+91 a^{12} x^n \left(b+c x^n\right)-364 a^{11} x^{2 n} \left(b+c x^n\right)^2+1001 a^{10} x^{3 n} \left(b+c x^n\right)^3-2002 a^9 x^{4 n} \left(b+c x^n\right)^4+3003 a^8 x^{5 n} \left(b+c x^n\right)^5-3432 a^7 x^{6 n} \left(b+c x^n\right)^6+3003 a^6 x^{7 n} \left(b+c x^n\right)^7-2002 a^5 x^{8 n} \left(b+c x^n\right)^8+1001 a^4 x^{9 n} \left(b+c x^n\right)^9-364 a^3 x^{10 n} \left(b+c x^n\right)^{10}+91 a^2 x^{11 n} \left(b+c x^n\right)^{11}-14 a x^{12 n} \left(b+c x^n\right)^{12}+x^{13 n} \left(b+c x^n\right)^{13}\right)$$

Problem 101: Result more than twice size of optimal antiderivative.

$$\int (b+2 c x) \left(b x+c x^2\right)^{13} \, dx$$

Optimal (type 1, 15 leaves, 1 step):

$$\frac{1}{14}\left(b x+c x^2\right)^{14}$$

Result (type 1, 172 leaves):

$$\frac{b^{14} x^{14}}{14}+b^{13} c x^{15}+\frac{13}{2} b^{12} c^2 x^{16}+26 b^{11} c^3 x^{17}+\frac{143}{2} b^{10} c^4 x^{18}+143 b^9 c^5 x^{19}+\frac{429}{2} b^8 c^6 x^{20}+\frac{1716}{7} b^7 c^7 x^{21}+\frac{429}{2} b^6 c^8 x^{22}+143 b^5 c^9 x^{23}+\frac{143}{2} b^4 c^{10} x^{24}+26 b^3 c^{11} x^{25}+\frac{13}{2} b^2 c^{12} x^{26}+b c^{13} x^{27}+\frac{c^{14} x^{28}}{14}$$

Problem 102: Result more than twice size of optimal antiderivative.

$$\int x\left(b+2 c x^2\right)\left(b x^2+c x^4\right)^{13} dx$$

Optimal (type 1, 16 leaves, 3 steps):

$$\frac{1}{28} x^{28}\left(b+c x^2\right)^{14}$$

Result (type 1, 182 leaves):

$$\frac{b^{14} x^{28}}{28}+\frac{1}{2} b^{13} c x^{30}+\frac{13}{4} b^{12} c^2 x^{32}+13 b^{11} c^3 x^{34}+\frac{143}{4} b^{10} c^4 x^{36}+\frac{143}{2} b^9 c^5 x^{38}+\frac{429}{4} b^8 c^6 x^{40}+\frac{858}{7} b^7 c^7 x^{42}+\frac{429}{4} b^6 c^8 x^{44}+\frac{143}{2} b^5 c^9 x^{46}+\frac{143}{4} b^4 c^{10} x^{48}+13 b^3 c^{11} x^{50}+\frac{13}{4} b^2 c^{12} x^{52}+\frac{1}{2} b c^{13} x^{54}+\frac{c^{14} x^{56}}{28}$$

Problem 103: Result more than twice size of optimal antiderivative.

$$\int x^2\left(b+2 c x^3\right)\left(b x^3+c x^6\right)^{13} dx$$

Optimal (type 1, 16 leaves, 3 steps):

$$\frac{1}{42} x^{42}\left(b+c x^3\right)^{14}$$

Result (type 1, 186 leaves):

$$\frac{b^{14} x^{42}}{42}+\frac{1}{3} b^{13} c x^{45}+\frac{13}{6} b^{12} c^2 x^{48}+\frac{26}{3} b^{11} c^3 x^{51}+\frac{143}{6} b^{10} c^4 x^{54}+\frac{143}{3} b^9 c^5 x^{57}+\frac{143}{2} b^8 c^6 x^{60}+\frac{572}{7} b^7 c^7 x^{63}+\frac{143}{2} b^6 c^8 x^{66}+\frac{143}{3} b^5 c^9 x^{69}+\frac{143}{6} b^4 c^{10} x^{72}+\frac{26}{3} b^3 c^{11} x^{75}+\frac{13}{6} b^2 c^{12} x^{78}+\frac{1}{3} b c^{13} x^{81}+\frac{c^{14} x^{84}}{42}$$

Problem 128: Result more than twice size of optimal antiderivative.

$$\int \frac{x^{-1+n} \left(b+2 c x^n\right)}{\left(b x^n+c x^{2 n}\right)^8} \, dx$$

Optimal (type 3, 21 leaves, 3 steps):

$$-\frac{x^{-7 n}}{7 n \left(b+c x^n\right)^7}$$

Result (type 3, 127 leaves):

$$-\frac{1}{7 b^{14} n \left(b+c x^n\right)^7} x^{-7 n} \left(b^{14}+1716 b^7 c^7 x^{7 n}+12012 b^6 c^8 x^{8 n}+36036 b^5 c^9 x^{9 n}+60060 b^4 c^{10} x^{10 n}+60060 b^3 c^{11} x^{11 n}+36036 b^2 c^{12} x^{12 n}+12012 b c^{13} x^{13 n}+1716 c^{14} x^{14 n}\right)$$

Problem 142: Result more than twice size of optimal antiderivative.

$$\int \frac{(f x)^m \left(d+e x^n\right)}{\left(a+b x^n+c x^{2 n}\right)^2} \, dx$$

Optimal (type 5, 374 leaves, 5 steps):

$$\frac{(f x)^{1+m} \left(b^2 d-2 a c d-a b e+c (b d-2 a e) x^n\right)}{a \left(b^2-4 a c\right) f n \left(a+b x^n+c x^{2 n}\right)}-\left(c \left((b d-2 a e) (1+m-n)-\frac{4 a c d (1+m-2 n)-b^2 d (1+m-n)+2 a b e n}{\sqrt{b^2-4 a c}}\right) (f x)^{1+m} \text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{2 c x^n}{b-\sqrt{b^2-4 a c}}\right]\right) \Big/ \left(a \left(b^2-4 a c\right) \left(b-\sqrt{b^2-4 a c}\right) f (1+m) n\right)-\left(c \left((b d-2 a e) (1+m-n)+\frac{4 a c d (1+m-2 n)-b^2 d (1+m-n)+2 a b e n}{\sqrt{b^2-4 a c}}\right) (f x)^{1+m} \text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{2 c x^n}{b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \left(a \left(b^2-4 a c\right) \left(b+\sqrt{b^2-4 a c}\right) f (1+m) n\right)$$

Result (type 5, 5363 leaves):

$$\frac{x (f x)^m \left(-b^2 d+2 a c d+a b e-b c d x^n+2 a c e x^n\right)}{a \left(-b^2+4 a c\right) n \left(a+b x^n+c x^{2 n}\right)}-$$

$$\left(b\,c\,d\,x^{1+n}\,(f\,x)^m\,(x^n)^{\frac{1+m}{n}-\frac{1+m+n}{n}}\left(-\frac{1}{\sqrt{b^2-4\,a\,c}}\left(\frac{x^n}{-\frac{-b-\sqrt{b^2-4\,a\,c}}{2\,c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\text{Hypergeometric2F1}\left[-\frac{1+m}{n},\right.\right.\right.$$

$$\left.-\frac{1+m}{n},1-\frac{1+m}{n},-\frac{-b-\sqrt{b^2-4\,a\,c}}{2\,c\left(-\frac{-b-\sqrt{b^2-4\,a\,c}}{2\,c}+x^n\right)}\right]+\frac{1}{\sqrt{b^2-4\,a\,c}}\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}$$

$$\left.\left.\text{Hypergeometric2F1}\left[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},-\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c\left(-\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c}+x^n\right)}\right]\right)\right)\Big/$$

$$\left(a\left(-b^2+4\,a\,c\right)(1+m)\right)+\left(2\,c\,e\,x^{1+n}\,(f\,x)^m\,(x^n)^{\frac{1+m}{n}-\frac{1+m+n}{n}}\right.$$

$$\left(-\frac{1}{\sqrt{b^2-4\,a\,c}}\left(\frac{x^n}{-\frac{-b-\sqrt{b^2-4\,a\,c}}{2\,c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\text{Hypergeometric2F1}\left[-\frac{1+m}{n},-\frac{1+m}{n},\right.\right.$$

$$\left.1-\frac{1+m}{n},-\frac{-b-\sqrt{b^2-4\,a\,c}}{2\,c\left(-\frac{-b-\sqrt{b^2-4\,a\,c}}{2\,c}+x^n\right)}\right]+\frac{1}{\sqrt{b^2-4\,a\,c}}\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}$$

$$\left.\left.\text{Hypergeometric2F1}\left[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},-\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c\left(-\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c}+x^n\right)}\right]\right)\right)\Big/$$

$$\left(\left(-b^2+4\,a\,c\right)(1+m)\right)+\left(b\,c\,d\,x^{1+n}\,(f\,x)^m\,(x^n)^{\frac{1+m}{n}-\frac{1+m+n}{n}}\right.$$

$$\left(-\frac{1}{\sqrt{b^2-4\,a\,c}}\left(\frac{x^n}{-\frac{-b-\sqrt{b^2-4\,a\,c}}{2\,c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\text{Hypergeometric2F1}\left[-\frac{1+m}{n},-\frac{1+m}{n},\right.\right.$$

$$1-\frac{1+m}{n},-\frac{-b-\sqrt{b^2-4ac}}{2c\left(-\frac{-b-\sqrt{b^2-4ac}}{2c}+x^n\right)}\Big]+\frac{1}{\sqrt{b^2-4ac}}\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}$$

$$\text{Hypergeometric2F1}\Big[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},-\frac{-b+\sqrt{b^2-4ac}}{2c\left(-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n\right)}\Big]\Bigg)\Bigg)\Bigg/$$

$$\left(a\left(-b^2+4ac\right)(1+m)\,n\right)-\Bigg(2\,c\,e\,x^{1+n}\,(f\,x)^m\,(x^n)^{\frac{1+m}{n}-\frac{1+m+n}{n}}$$

$$\Bigg(-\frac{1}{\sqrt{b^2-4ac}}\left(\frac{x^n}{-\frac{-b-\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\text{Hypergeometric2F1}\Big[-\frac{1+m}{n},-\frac{1+m}{n},$$

$$1-\frac{1+m}{n},-\frac{-b-\sqrt{b^2-4ac}}{2c\left(-\frac{-b-\sqrt{b^2-4ac}}{2c}+x^n\right)}\Big]+\frac{1}{\sqrt{b^2-4ac}}\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}$$

$$\text{Hypergeometric2F1}\Big[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},-\frac{-b+\sqrt{b^2-4ac}}{2c\left(-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n\right)}\Big]\Bigg)\Bigg)\Bigg/$$

$$\left(\left(-b^2+4ac\right)(1+m)\,n\right)+\Bigg(b\,c\,d\,m\,x^{1+n}\,(f\,x)^m\,(x^n)^{\frac{1+m}{n}-\frac{1+m+n}{n}}$$

$$\Bigg(-\frac{1}{\sqrt{b^2-4ac}}\left(\frac{x^n}{-\frac{-b-\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\text{Hypergeometric2F1}\Big[-\frac{1+m}{n},-\frac{1+m}{n},$$

$$1-\frac{1+m}{n},-\frac{-b-\sqrt{b^2-4ac}}{2c\left(-\frac{-b-\sqrt{b^2-4ac}}{2c}+x^n\right)}\Big]+\frac{1}{\sqrt{b^2-4ac}}\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}$$

$$\text{Hypergeometric2F1}\left[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},-\frac{-b+\sqrt{b^2-4ac}}{2c\left(-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n\right)}\right]\Bigg)\Bigg)\Bigg/$$

$$\left(a\left(-b^2+4ac\right)(1+m)\,n\right)-\Bigg(2\,c\,e\,m\,x^{1+n}\,(f x)^m\,\left(x^n\right)^{\frac{1+m}{n}-\frac{1+m+n}{n}}$$

$$\Bigg(-\frac{1}{\sqrt{b^2-4ac}}\left(\frac{x^n}{-\frac{-b-\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\text{Hypergeometric2F1}\Bigg[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},$$

$$-\frac{-b-\sqrt{b^2-4ac}}{2c\left(-\frac{-b-\sqrt{b^2-4ac}}{2c}+x^n\right)}\Bigg]+\frac{1}{\sqrt{b^2-4ac}}\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\text{Hypergeometric2F1}\Bigg[$$

$$-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},-\frac{-b+\sqrt{b^2-4ac}}{2c\left(-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n\right)}\Bigg]\Bigg)\Bigg)\Bigg/\left(\left(-b^2+4ac\right)(1+m)\,n\right)+$$

$$\Bigg(b^2\,d\,x\,(f x)^m\Bigg(\Bigg(1-\left(\frac{x^n}{-\frac{-b-\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\text{Hypergeometric2F1}\Bigg[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},$$

$$-\frac{-b-\sqrt{b^2-4ac}}{2c\left(-\frac{-b-\sqrt{b^2-4ac}}{2c}+x^n\right)}\Bigg]\Bigg)\Bigg/\left(\frac{b\left(-b-\sqrt{b^2-4ac}\right)}{2c}+\frac{\left(-b-\sqrt{b^2-4ac}\right)^2}{2c}\right)+$$

$$\Bigg(1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\text{Hypergeometric2F1}\Bigg[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},$$

$$-\frac{-b+\sqrt{b^2-4ac}}{2c\left(-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n\right)}\Bigg]\Bigg)\Bigg/\left(\frac{b\left(-b+\sqrt{b^2-4ac}\right)}{2c}+\frac{\left(-b+\sqrt{b^2-4ac}\right)^2}{2c}\right)\Bigg)\Bigg)\Bigg/$$

$$\left(a\left(-b^2+4ac\right)(1+m)\right)-\left(4cdx(fx)^m\left(\left(\left(1-\left(\frac{x^n}{-\frac{-b-\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\right.\right.\right.\right.$$

$$\left.\text{Hypergeometric2F1}\left[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},-\frac{-b-\sqrt{b^2-4ac}}{2c\left(-\frac{-b-\sqrt{b^2-4ac}}{2c}+x^n\right)}\right]\right)\Bigg/$$

$$\left(\frac{b\left(-b-\sqrt{b^2-4ac}\right)}{2c}+\frac{\left(-b-\sqrt{b^2-4ac}\right)^2}{2c}\right)+\left(1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\right.$$

$$\left.\text{Hypergeometric2F1}\left[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},-\frac{-b+\sqrt{b^2-4ac}}{2c\left(-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n\right)}\right]\right)\Bigg/$$

$$\left.\left.\left(\frac{b\left(-b+\sqrt{b^2-4ac}\right)}{2c}+\frac{\left(-b+\sqrt{b^2-4ac}\right)^2}{2c}\right)\right)\right)\Bigg/\left(\left(-b^2+4ac\right)(1+m)\right)-$$

$$\left(b^2dx(fx)^m\left(\left(1-\left(\frac{x^n}{-\frac{-b-\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\text{Hypergeometric2F1}\left[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},\right.\right.\right.\right.$$

$$\left.\left.-\frac{-b-\sqrt{b^2-4ac}}{2c\left(-\frac{-b-\sqrt{b^2-4ac}}{2c}+x^n\right)}\right]\right)\Bigg/\left(\frac{b\left(-b-\sqrt{b^2-4ac}\right)}{2c}+\frac{\left(-b-\sqrt{b^2-4ac}\right)^2}{2c}\right)+$$

$$\left(1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\text{Hypergeometric2F1}\left[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},\right.\right.$$

$$\left.\left.\left.\left.-\frac{-b+\sqrt{b^2-4ac}}{2c\left(-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n\right)}\right]\right)\Bigg/\left(\frac{b\left(-b+\sqrt{b^2-4ac}\right)}{2c}+\frac{\left(-b+\sqrt{b^2-4ac}\right)^2}{2c}\right)\right)\right)\Bigg/$$

$$\left(a\left(-b^2+4ac\right)(1+m)\,n\right)+\left(2cdx\,(fx)^m\left(\left(\left(1-\left(\frac{x^n}{-\frac{-b-\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\right.\right.\right.\right.$$

$$\left.\text{Hypergeometric2F1}\left[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},-\frac{-b-\sqrt{b^2-4ac}}{2c\left(-\frac{-b-\sqrt{b^2-4ac}}{2c}+x^n\right)}\right]\right)\Big/$$

$$\left(\frac{b\left(-b-\sqrt{b^2-4ac}\right)}{2c}+\frac{\left(-b-\sqrt{b^2-4ac}\right)^2}{2c}\right)+\left(1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\right.$$

$$\left.\text{Hypergeometric2F1}\left[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},-\frac{-b+\sqrt{b^2-4ac}}{2c\left(-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n\right)}\right]\right)\Big/$$

$$\left.\left.\left(\frac{b\left(-b+\sqrt{b^2-4ac}\right)}{2c}+\frac{\left(-b+\sqrt{b^2-4ac}\right)^2}{2c}\right)\right)\right)\Big/\left(\left(-b^2+4ac\right)(1+m)\,n\right)+$$

$$\left(bex\,(fx)^m\left(\left(\left(1-\left(\frac{x^n}{-\frac{-b-\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\text{Hypergeometric2F1}\left[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},\right.\right.\right.\right.\right.$$

$$\left.\left.-\frac{-b-\sqrt{b^2-4ac}}{2c\left(-\frac{-b-\sqrt{b^2-4ac}}{2c}+x^n\right)}\right]\right)\Big/\left(\frac{b\left(-b-\sqrt{b^2-4ac}\right)}{2c}+\frac{\left(-b-\sqrt{b^2-4ac}\right)^2}{2c}\right)+$$

$$\left(1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\text{Hypergeometric2F1}\left[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},\right.\right.$$

$$\left.\left.\left.-\frac{-b+\sqrt{b^2-4ac}}{2c\left(-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n\right)}\right]\right)\Big/\left(\frac{b\left(-b+\sqrt{b^2-4ac}\right)}{2c}+\frac{\left(-b+\sqrt{b^2-4ac}\right)^2}{2c}\right)\right)\right)\Big/$$

$$\left(\left(-b^2+4\,a\,c\right)\,(1+m)\,n\right)-\left(b^2\,d\,m\,x\,(f\,x)^m\left(\left(\left(1-\left(\frac{x^n}{-\frac{-b-\sqrt{b^2-4\,a\,c}}{2\,c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\right.\right.\right.\right.$$

$$\left.\text{Hypergeometric2F1}\left[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},-\frac{-b-\sqrt{b^2-4\,a\,c}}{2\,c\left(-\frac{-b-\sqrt{b^2-4\,a\,c}}{2\,c}+x^n\right)}\right]\right)\Big/$$

$$\left(\frac{b\left(-b-\sqrt{b^2-4\,a\,c}\right)}{2\,c}+\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)^2}{2\,c}\right)+\left(1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\right.$$

$$\left.\text{Hypergeometric2F1}\left[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},-\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c\left(-\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c}+x^n\right)}\right]\right)\Big/$$

$$\left.\left.\left(\frac{b\left(-b+\sqrt{b^2-4\,a\,c}\right)}{2\,c}+\frac{\left(-b+\sqrt{b^2-4\,a\,c}\right)^2}{2\,c}\right)\right)\right)\Big/\left(a\left(-b^2+4\,a\,c\right)\,(1+m)\,n\right)+$$

$$\left(2\,c\,d\,m\,x\,(f\,x)^m\left(\left(\left(1-\left(\frac{x^n}{-\frac{-b-\sqrt{b^2-4\,a\,c}}{2\,c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\text{Hypergeometric2F1}\left[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},\right.\right.\right.\right.\right.$$

$$\left.\left.-\frac{-b-\sqrt{b^2-4\,a\,c}}{2\,c\left(-\frac{-b-\sqrt{b^2-4\,a\,c}}{2\,c}+x^n\right)}\right]\right)\Big/\left(\frac{b\left(-b-\sqrt{b^2-4\,a\,c}\right)}{2\,c}+\frac{\left(-b-\sqrt{b^2-4\,a\,c}\right)^2}{2\,c}\right)+$$

$$\left(1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\text{Hypergeometric2F1}\left[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},\right.\right.$$

$$\left.\left.\left.\left.-\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c\left(-\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c}+x^n\right)}\right]\right)\Big/\left(\frac{b\left(-b+\sqrt{b^2-4\,a\,c}\right)}{2\,c}+\frac{\left(-b+\sqrt{b^2-4\,a\,c}\right)^2}{2\,c}\right)\right)\right)\Big/$$

$$\left(\left(-b^2+4 a c\right) (1+m) n\right)+\left(b e m x (f x)^m \left(\left(1-\left(\frac{x^n}{-\frac{-b-\sqrt{b^2-4 a c}}{2 c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\right.\right.\right.$$

$$\left.\text{Hypergeometric2F1}\left[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},-\frac{-b-\sqrt{b^2-4 a c}}{2 c\left(-\frac{-b-\sqrt{b^2-4 a c}}{2 c}+x^n\right)}\right]\right)\Bigg/$$

$$\left(\frac{b\left(-b-\sqrt{b^2-4 a c}\right)}{2 c}+\frac{\left(-b-\sqrt{b^2-4 a c}\right)^2}{2 c}\right)+\left(1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4 a c}}{2 c}+x^n}\right)^{-\frac{1}{n}-\frac{m}{n}}\right.$$

$$\left.\text{Hypergeometric2F1}\left[-\frac{1+m}{n},-\frac{1+m}{n},1-\frac{1+m}{n},-\frac{-b+\sqrt{b^2-4 a c}}{2 c\left(-\frac{-b+\sqrt{b^2-4 a c}}{2 c}+x^n\right)}\right]\right)\Bigg/$$

$$\left.\left.\left(\frac{b\left(-b+\sqrt{b^2-4 a c}\right)}{2 c}+\frac{\left(-b+\sqrt{b^2-4 a c}\right)^2}{2 c}\right)\right)\right)\Bigg/\left(\left(-b^2+4 a c\right) (1+m) n\right)$$

Problem 143: Result more than twice size of optimal antiderivative.

$$\int \frac{(f x)^m \left(d+e x^n\right)}{\left(a+b x^n+c x^{2 n}\right)^3} \, dx$$

Optimal (type 5, 816 leaves, 6 steps):

$$\frac{(f x)^{1+m} \left(b^2 d-2 a c d-a b e+c (b d-2 a e) x^n\right)}{2 a \left(b^2-4 a c\right) f n \left(a+b x^n+c x^{2 n}\right)^2}+$$
$$\left((f x)^{1+m} \left(\left(b^2-2 a c\right) (a b e (1+m)+2 a c d (1+m-4 n)-b^2 d (1+m-2 n))+\right.\right.$$
$$a b c (b d-2 a e) (1+m-3 n)+$$
$$\left.\left.c \left(a b^2 e (1+m)+2 a b c d (2+2 m-7 n)-4 a^2 c e (1+m-3 n)-b^3 d (1+m-2 n)\right) x^n\right)\right)\Big/$$
$$\left(2 a^2 \left(b^2-4 a c\right)^2 f n^2 \left(a+b x^n+c x^{2 n}\right)\right)-$$
$$\left(c \left(\left(a b^2 e (1+m)+2 a b c d (2+2 m-7 n)-4 a^2 c e (1+m-3 n)-b^3 d (1+m-2 n)\right) (1+m-n)+\right.\right.$$
$$\frac{1}{\sqrt{b^2-4 a c}}\left(a b^3 e (1+m) (1+m-n)-4 a^2 b c e \left(1+m^2+m (2-n)-n-3 n^2\right)-\right.$$
$$b^4 d \left(1+m^2+m (2-3 n)-3 n+2 n^2\right)+6 a b^2 c d \left(1+m^2+m (2-4 n)-4 n+3 n^2\right)-$$
$$\left.\left.8 a^2 c^2 d \left(1+m^2+m (2-6 n)-6 n+8 n^2\right)\right)\right)$$
$$\left.(f x)^{1+m} \text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{2 c x^n}{b-\sqrt{b^2-4 a c}}\right]\right)\Big/$$
$$\left(2 a^2 \left(b^2-4 a c\right)^2 \left(b-\sqrt{b^2-4 a c}\right) f (1+m) n^2\right)-$$
$$\left(c \left(\left(a b^2 e (1+m)+2 a b c d (2+2 m-7 n)-4 a^2 c e (1+m-3 n)-b^3 d (1+m-2 n)\right) (1+m-n)-\right.\right.$$
$$\frac{1}{\sqrt{b^2-4 a c}}\left(a b^3 e (1+m) (1+m-n)-4 a^2 b c e \left(1+m^2+m (2-n)-n-3 n^2\right)-\right.$$
$$b^4 d \left(1+m^2+m (2-3 n)-3 n+2 n^2\right)+6 a b^2 c d \left(1+m^2+m (2-4 n)-4 n+3 n^2\right)-$$
$$\left.\left.8 a^2 c^2 d \left(1+m^2+m (2-6 n)-6 n+8 n^2\right)\right)\right)$$
$$\left.(f x)^{1+m} \text{Hypergeometric2F1}\left[1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{2 c x^n}{b+\sqrt{b^2-4 a c}}\right]\right)\Big/$$
$$\left(2 a^2 \left(b^2-4 a c\right)^2 \left(b+\sqrt{b^2-4 a c}\right) f (1+m) n^2\right)$$

Result (type 5, 20 515 leaves) : Display of huge result suppressed!

Problem 145: Unable to integrate problem.

$$\int \frac{(f x)^m \left(d+e x^n\right)^q}{a+b x^n+c x^{2 n}} \, dx$$

Optimal (type 6, 245 leaves, 5 steps) :

$$\left(2\,c\,(f\,x)^{1+m}\,(d+e\,x^n)^q\,\left(1+\frac{e\,x^n}{d}\right)^{-q}\,\text{AppellF1}\left[\frac{1+m}{n},\,1,\,-q,\,\frac{1+m+n}{n},\,-\frac{2\,c\,x^n}{b-\sqrt{b^2-4\,a\,c}},\,-\frac{e\,x^n}{d}\right]\right)\Big/\left(\sqrt{b^2-4\,a\,c}\,\left(b-\sqrt{b^2-4\,a\,c}\right)\,f\,(1+m)\right)-\left(2\,c\,(f\,x)^{1+m}\,(d+e\,x^n)^q\,\left(1+\frac{e\,x^n}{d}\right)^{-q}\,\text{AppellF1}\left[\frac{1+m}{n},\,1,\,-q,\,\frac{1+m+n}{n},\,-\frac{2\,c\,x^n}{b+\sqrt{b^2-4\,a\,c}},\,-\frac{e\,x^n}{d}\right]\right)\Big/\left(\sqrt{b^2-4\,a\,c}\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,f\,(1+m)\right)$$

Result (type 8, 33 leaves):

$$\int\frac{(f\,x)^m\,(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Problem 146: Unable to integrate problem.

$$\int\frac{x^2\,(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Optimal (type 6, 210 leaves, 5 steps):

$$-\left(\left(2\,c\,x^3\,(d+e\,x^n)^q\,\left(1+\frac{e\,x^n}{d}\right)^{-q}\,\text{AppellF1}\left[\frac{3}{n},\,1,\,-q,\,\frac{3+n}{n},\,-\frac{2\,c\,x^n}{b-\sqrt{b^2-4\,a\,c}},\,-\frac{e\,x^n}{d}\right]\right)\Big/\left(3\,\left(b^2-4\,a\,c-b\,\sqrt{b^2-4\,a\,c}\right)\right)\right)-\left(2\,c\,x^3\,(d+e\,x^n)^q\,\left(1+\frac{e\,x^n}{d}\right)^{-q}\,\text{AppellF1}\left[\frac{3}{n},\,1,\,-q,\,\frac{3+n}{n},\,-\frac{2\,c\,x^n}{b+\sqrt{b^2-4\,a\,c}},\,-\frac{e\,x^n}{d}\right]\right)\Big/\left(3\,\left(b^2-4\,a\,c+b\,\sqrt{b^2-4\,a\,c}\right)\right)$$

Result (type 8, 31 leaves):

$$\int\frac{x^2\,(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Problem 147: Unable to integrate problem.

$$\int\frac{x\,(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Optimal (type 6, 206 leaves, 5 steps):

$$-\left(\left(c\,x^2\,(d+e\,x^n)^q\left(1+\frac{e\,x^n}{d}\right)^{-q}\text{AppellF1}\left[\frac{2}{n},1,-q,\frac{2+n}{n},-\frac{2\,c\,x^n}{b-\sqrt{b^2-4\,a\,c}},-\frac{e\,x^n}{d}\right]\right)\Big/\left(b^2-4\,a\,c-b\,\sqrt{b^2-4\,a\,c}\right)\right)-\left(c\,x^2\,(d+e\,x^n)^q\left(1+\frac{e\,x^n}{d}\right)^{-q}\text{AppellF1}\left[\frac{2}{n},1,-q,\frac{2+n}{n},-\frac{2\,c\,x^n}{b+\sqrt{b^2-4\,a\,c}},-\frac{e\,x^n}{d}\right]\right)\Big/\left(b^2-4\,a\,c+b\,\sqrt{b^2-4\,a\,c}\right)$$

Result (type 8, 29 leaves):

$$\int\frac{x\,(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Problem 148: Unable to integrate problem.

$$\int\frac{(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Optimal (type 6, 194 leaves, 5 steps):

$$-\left(\left(2\,c\,x\,(d+e\,x^n)^q\left(1+\frac{e\,x^n}{d}\right)^{-q}\text{AppellF1}\left[\frac{1}{n},1,-q,1+\frac{1}{n},-\frac{2\,c\,x^n}{b-\sqrt{b^2-4\,a\,c}},-\frac{e\,x^n}{d}\right]\right)\Big/\left(b^2-4\,a\,c-b\,\sqrt{b^2-4\,a\,c}\right)\right)-\left(2\,c\,x\,(d+e\,x^n)^q\left(1+\frac{e\,x^n}{d}\right)^{-q}\text{AppellF1}\left[\frac{1}{n},1,-q,1+\frac{1}{n},-\frac{2\,c\,x^n}{b+\sqrt{b^2-4\,a\,c}},-\frac{e\,x^n}{d}\right]\right)\Big/\left(b^2-4\,a\,c+b\,\sqrt{b^2-4\,a\,c}\right)$$

Result (type 8, 28 leaves):

$$\int\frac{(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Problem 149: Unable to integrate problem.

$$\int\frac{(d+e\,x^n)^q}{x\,(a+b\,x^n+c\,x^{2\,n})}\,dx$$

Optimal (type 5, 263 leaves, 8 steps):

$$\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+e x^n)^{1+q}\,\text{Hypergeometric2F1}\left[1,1+q,2+q,\frac{2c(d+e x^n)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right]\right)\Big/\left(a\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)n(1+q)\right)+\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+e x^n)^{1+q}\,\text{Hypergeometric2F1}\left[1,1+q,2+q,\frac{2c(d+e x^n)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right]\right)\Big/\left(a\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)n(1+q)\right)-\frac{(d+e x^n)^{1+q}\,\text{Hypergeometric2F1}\left[1,1+q,2+q,1+\frac{e x^n}{d}\right]}{a d n(1+q)}$$

Result (type 8, 31 leaves):

$$\int\frac{(d+e x^n)^q}{x\,(a+b x^n+c x^{2n})}\,dx$$

Problem 150: Unable to integrate problem.

$$\int\frac{(d+e x^n)^q}{x^2\,(a+b x^n+c x^{2n})}\,dx$$

Optimal (type 6, 212 leaves, 5 steps):

$$\left(2c(d+e x^n)^q\left(1+\frac{e x^n}{d}\right)^{-q}\text{AppellF1}\left[-\frac{1}{n},1,-q,-\frac{1-n}{n},-\frac{2c x^n}{b-\sqrt{b^2-4ac}},-\frac{e x^n}{d}\right]\right)\Big/\left(\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x\right)+\left(2c(d+e x^n)^q\left(1+\frac{e x^n}{d}\right)^{-q}\text{AppellF1}\left[-\frac{1}{n},1,-q,-\frac{1-n}{n},-\frac{2c x^n}{b+\sqrt{b^2-4ac}},-\frac{e x^n}{d}\right]\right)\Big/\left(\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x\right)$$

Result (type 8, 31 leaves):

$$\int\frac{(d+e x^n)^q}{x^2\,(a+b x^n+c x^{2n})}\,dx$$

Problem 151: Unable to integrate problem.

$$\int\frac{(d+e x^n)^q}{x^3\,(a+b x^n+c x^{2n})}\,dx$$

Optimal (type 6, 210 leaves, 5 steps):

$$\left(c\left(d+e x^n\right)^q\left(1+\frac{e x^n}{d}\right)^{-q}\text{AppellF1}\left[-\frac{2}{n},1,-q,-\frac{2-n}{n},-\frac{2 c x^n}{b-\sqrt{b^2-4 a c}},-\frac{e x^n}{d}\right]\right)\Big/\left(\left(b^2-4 a c-b\sqrt{b^2-4 a c}\right)x^2\right)+\left(c\left(d+e x^n\right)^q\left(1+\frac{e x^n}{d}\right)^{-q}\text{AppellF1}\left[-\frac{2}{n},1,-q,-\frac{2-n}{n},-\frac{2 c x^n}{b+\sqrt{b^2-4 a c}},-\frac{e x^n}{d}\right]\right)\Big/\left(\left(b^2-4 a c+b\sqrt{b^2-4 a c}\right)x^2\right)$$

Result (type 8, 31 leaves):

$$\int\frac{\left(d+e x^n\right)^q}{x^3\left(a+b x^n+c x^{2 n}\right)}\,dx$$

Problem 152: Result more than twice size of optimal antiderivative.

$$\int(f x)^m\left(d+e x^n\right)^2\left(a+b x^n+c x^{2 n}\right)^p\,dx$$

Optimal (type 6, 498 leaves, 10 steps):

$$\frac{1}{f(1+m)}d^2(f x)^{1+m}\left(1+\frac{2 c x^n}{b-\sqrt{b^2-4 a c}}\right)^{-p}\left(1+\frac{2 c x^n}{b+\sqrt{b^2-4 a c}}\right)^{-p}\left(a+b x^n+c x^{2 n}\right)^p\text{AppellF1}\left[\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2 c x^n}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^n}{b+\sqrt{b^2-4 a c}}\right]+\frac{1}{1+m+n}2 d e x^{1+n}(f x)^m\left(1+\frac{2 c x^n}{b-\sqrt{b^2-4 a c}}\right)^{-p}\left(1+\frac{2 c x^n}{b+\sqrt{b^2-4 a c}}\right)^{-p}\left(a+b x^n+c x^{2 n}\right)^p\text{AppellF1}\left[\frac{1+m+n}{n},-p,-p,\frac{1+m+2 n}{n},-\frac{2 c x^n}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^n}{b+\sqrt{b^2-4 a c}}\right]+\frac{1}{1+m+2 n}e^2 x^{1+2 n}(f x)^m\left(1+\frac{2 c x^n}{b-\sqrt{b^2-4 a c}}\right)^{-p}\left(1+\frac{2 c x^n}{b+\sqrt{b^2-4 a c}}\right)^{-p}\left(a+b x^n+c x^{2 n}\right)^p\text{AppellF1}\left[\frac{1+m+2 n}{n},-p,-p,\frac{1+m+3 n}{n},-\frac{2 c x^n}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^n}{b+\sqrt{b^2-4 a c}}\right]$$

Result (type 6, 1762 leaves):

$$-\left(\left(2^{-1-p}\left(b+\sqrt{b^2-4 a c}\right)d^2(1+m+n)\,x\,(f x)^m\left(\frac{b-\sqrt{b^2-4 a c}}{2 c}+x^n\right)^{-p}\left(-b+\sqrt{b^2-4 a c}-2 c x^n\right)\left(\frac{b-\sqrt{b^2-4 a c}+2 c x^n}{c}\right)^p\left(-2 a+\left(-b+\sqrt{b^2-4 a c}\right)x^n\right)^2\left(a+x^n\left(b+c x^n\right)\right)^{-1+p}\text{AppellF1}\left[\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2 c x^n}{b+\sqrt{b^2-4 a c}},\frac{2 c x^n}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/\left(\left(-b+\sqrt{b^2-4 a c}\right)(1+m)\left(b+\sqrt{b^2-4 a c}+2 c x^n\right)\right.\right.$$

$$\left(-2\,a\,(1+m+n)\,\text{AppellF1}\left[\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2\,c\,x^n}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^n}{-b+\sqrt{b^2-4\,a\,c}}\right]+\right.$$
$$n\,p\,x^n\left(\left(-b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{1+m+n}{n},1-p,-p,\frac{1+m+2\,n}{n},\right.\right.$$
$$\left.-\frac{2\,c\,x^n}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^n}{-b+\sqrt{b^2-4\,a\,c}}\right]-\left(b+\sqrt{b^2-4\,a\,c}\right)$$
$$\left.\left.\left.\left.\text{AppellF1}\left[\frac{1+m+n}{n},-p,1-p,\frac{1+m+2\,n}{n},-\frac{2\,c\,x^n}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^n}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)\right)+$$
$$\left(2^{-2\,p}\left(b+\sqrt{b^2-4\,a\,c}\right)d\,e\,(1+m+2\,n)\,x^{1+n}\,(f\,x)^m\left(-\frac{-b-\sqrt{b^2-4\,a\,c}}{2\,c}+x^n\right)^{-p}\right.$$
$$\left(-\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c}+x^n\right)^{-p}$$
$$\left(\frac{-b+\sqrt{b^2-4\,a\,c}-2\,c\,x^n}{-b+\sqrt{b^2-4\,a\,c}}\right)^{-p}$$
$$\left(\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x^n}{c}\right)^{p}$$
$$\left(\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x^n}{b-\sqrt{b^2-4\,a\,c}}\right)^{1+p}\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^n}{c}\right)^{p}$$
$$\left(2\,a+\left(b-\sqrt{b^2-4\,a\,c}\right)x^n\right)^2\left(a+b\,x^n+c\,x^{2\,n}\right)^p$$
$$\left.\text{AppellF1}\left[\frac{1+m+n}{n},-p,-p,1+\frac{1+m+n}{n},\frac{2\,c\,x^n}{-b-\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^n}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left((1+m+n)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^n\right)\left(a+x^n\,(b+c\,x^n)\right)\right.$$
$$\left(2\,a\,(1+m+2\,n)\,\text{AppellF1}\left[\frac{1+m+n}{n},-p,-p,\frac{1+m+2\,n}{n},-\frac{2\,c\,x^n}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^n}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$
$$n\,p\,x^n\left(\left(-b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\left[\frac{1+m+2\,n}{n},1-p,-p,\right.\right.$$
$$\left.\frac{1+m+3\,n}{n},-\frac{2\,c\,x^n}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^n}{-b+\sqrt{b^2-4\,a\,c}}\right]-\left(b+\sqrt{b^2-4\,a\,c}\right)$$
$$\left.\left.\left.\left.\text{AppellF1}\left[\frac{1+m+2\,n}{n},-p,1-p,\frac{1+m+3\,n}{n},-\frac{2\,c\,x^n}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^n}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\right)\right)+$$
$$\left(2^{-1-p}\,c\left(b+\sqrt{b^2-4\,a\,c}\right)e^2\,(1+m+3\,n)\,x^{1+2\,n}\,(f\,x)^m\left(\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+x^n\right)^{-p}\right.$$
$$\left(\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x^n}{c}\right)^{1+p}$$
$$\left(-2\,a+\left(-b+\sqrt{b^2-4\,a\,c}\right)x^n\right)^2\left(a+x^n\,(b+c\,x^n)\right)^{-1+p}$$

$$\text{AppellF1}\left[\frac{1+m+2n}{n},-p,-p,\frac{1+m+3n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right]\Bigg/$$
$$\Bigg(\left(-b+\sqrt{b^2-4ac}\right)(1+m+2n)\left(b+\sqrt{b^2-4ac}+2cx^n\right)$$
$$\Bigg(-2a(1+m+3n)\,\text{AppellF1}\left[\frac{1+m+2n}{n},-p,-p,\frac{1+m+3n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\right.$$
$$\left.\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right]+n\,p\,x^n\Bigg(\left(-b+\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{1+m+3n}{n},1-p,\right.$$
$$\left.-p,\frac{1+m+4n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right]-\left(b+\sqrt{b^2-4ac}\right)$$
$$\text{AppellF1}\left[\frac{1+m+3n}{n},-p,1-p,\frac{1+m+4n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Bigg)\Bigg)$$

Problem 153: Result more than twice size of optimal antiderivative.

$$\int (f x)^m\left(d+e x^n\right)\left(a+b x^n+c x^{2n}\right)^p\,dx$$

Optimal (type 6, 323 leaves, 7 steps):

$$\frac{1}{f(1+m)}d\,(f x)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p$$
$$\text{AppellF1}\left[\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right]+$$
$$\frac{1}{1+m+n}e\,x^{1+n}(f x)^m\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p$$
$$\text{AppellF1}\left[\frac{1+m+n}{n},-p,-p,\frac{1+m+2n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right]$$

Result (type 6, 1217 leaves):

$$-\Bigg(\Bigg(2^{-1-p}\left(b+\sqrt{b^2-4ac}\right)d\,(1+m+n)\,x\,(f x)^m\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^n\right)^{-p}\left(-b+\sqrt{b^2-4ac}-2cx^n\right)$$
$$\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{c}\right)^p\left(-2a+\left(-b+\sqrt{b^2-4ac}\right)x^n\right)^2\left(a+x^n\left(b+cx^n\right)\right)^{-1+p}$$
$$\text{AppellF1}\left[\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right]\Bigg)\Bigg/$$
$$\Bigg(\left(-b+\sqrt{b^2-4ac}\right)(1+m)\left(b+\sqrt{b^2-4ac}+2cx^n\right)$$
$$\Bigg(-2a(1+m+n)\,\text{AppellF1}\left[\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right]+$$
$$n\,p\,x^n\Bigg(\left(-b+\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{1+m+n}{n},1-p,-p,\frac{1+m+2n}{n},\right.$$

$$-\frac{2\,c\,x^n}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^n}{-b+\sqrt{b^2-4\,a\,c}}\Big]-\left(b+\sqrt{b^2-4\,a\,c}\right)$$

$$\text{AppellF1}\Big[\frac{1+m+n}{n},\,-p,\,1-p,\,\frac{1+m+2\,n}{n},\,-\frac{2\,c\,x^n}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^n}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg)\Bigg)+$$

$$\Bigg(2^{-1-2\,p}\left(b+\sqrt{b^2-4\,a\,c}\right)e\,(1+m+2\,n)\,x^{1+n}\,(f\,x)^m\left(-\frac{-b-\sqrt{b^2-4\,a\,c}}{2\,c}+x^n\right)^{-p}$$

$$\left(-\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,c}+x^n\right)^{-p}$$

$$\left(\frac{-b+\sqrt{b^2-4\,a\,c}-2\,c\,x^n}{-b+\sqrt{b^2-4\,a\,c}}\right)^{-p}$$

$$\left(\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x^n}{c}\right)^{p}$$

$$\left(\frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x^n}{b-\sqrt{b^2-4\,a\,c}}\right)^{1+p}\left(\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^n}{c}\right)^{p}$$

$$\left(2\,a+\left(b-\sqrt{b^2-4\,a\,c}\right)x^n\right)^2\left(a+b\,x^n+c\,x^{2\,n}\right)^p$$

$$\text{AppellF1}\Big[\frac{1+m+n}{n},\,-p,\,-p,\,1+\frac{1+m+n}{n},\,\frac{2\,c\,x^n}{-b-\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^n}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Big/$$

$$\Bigg((1+m+n)\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^n\right)\left(a+x^n\,(b+c\,x^n)\right)$$

$$\Bigg(2\,a\,(1+m+2\,n)\,\text{AppellF1}\Big[\frac{1+m+n}{n},\,-p,\,-p,\,\frac{1+m+2\,n}{n},\,-\frac{2\,c\,x^n}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^n}{-b+\sqrt{b^2-4\,a\,c}}\Big]-$$

$$n\,p\,x^n\Bigg(\left(-b+\sqrt{b^2-4\,a\,c}\right)\text{AppellF1}\Big[\frac{1+m+2\,n}{n},\,1-p,\,-p,$$

$$\frac{1+m+3\,n}{n},\,-\frac{2\,c\,x^n}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^n}{-b+\sqrt{b^2-4\,a\,c}}\Big]-\left(b+\sqrt{b^2-4\,a\,c}\right)$$

$$\text{AppellF1}\Big[\frac{1+m+2\,n}{n},\,-p,\,1-p,\,\frac{1+m+3\,n}{n},\,-\frac{2\,c\,x^n}{b+\sqrt{b^2-4\,a\,c}},\,\frac{2\,c\,x^n}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg)$$

Problem 154: Result more than twice size of optimal antiderivative.

$$\int (f\,x)^m\left(a+b\,x^n+c\,x^{2\,n}\right)^p\,dx$$

Optimal (type 6, 158 leaves, 2 steps):

$$\frac{1}{f\,(1+m)}(f\,x)^{1+m}\left(1+\frac{2\,c\,x^n}{b-\sqrt{b^2-4\,a\,c}}\right)^{-p}\left(1+\frac{2\,c\,x^n}{b+\sqrt{b^2-4\,a\,c}}\right)^{-p}\left(a+b\,x^n+c\,x^{2\,n}\right)^p$$

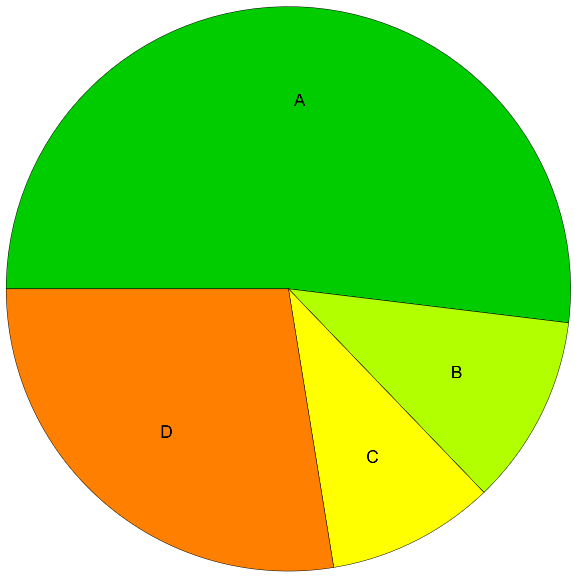
$$\text{AppellF1}\Big[\frac{1+m}{n},\,-p,\,-p,\,\frac{1+m+n}{n},\,-\frac{2\,c\,x^n}{b-\sqrt{b^2-4\,a\,c}},\,-\frac{2\,c\,x^n}{b+\sqrt{b^2-4\,a\,c}}\Big]$$

Result (type 6, 534 leaves) :

$$-\left(\left(2^{-1-p}\left(b+\sqrt{b^2-4ac}\right)(1+m+n)\,x\,(f x)^m\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^n\right)^{-p}\left(-b+\sqrt{b^2-4ac}-2cx^n\right)\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{c}\right)^p\left(-2a+\left(-b+\sqrt{b^2-4ac}\right)x^n\right)^2\left(a+x^n\left(b+cx^n\right)\right)^{-1+p}\text{AppellF1}\left[\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right]\right)\Big/\left(\left(-b+\sqrt{b^2-4ac}\right)(1+m)\left(b+\sqrt{b^2-4ac}+2cx^n\right)\left(-2a(1+m+n)\,\text{AppellF1}\left[\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right]+n\,p\,x^n\left(\left(-b+\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{1+m+n}{n},1-p,-p,\frac{1+m+2n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right]-\left(b+\sqrt{b^2-4ac}\right)\text{AppellF1}\left[\frac{1+m+n}{n},-p,1-p,\frac{1+m+2n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right]\right)\right)\right)\right)$$

Summary of Integration Test Results

156 integration problems



A - 81 optimal antiderivatives

B - 17 more than twice size of optimal antiderivatives

C - 15 unnecessarily complex antiderivatives

D - 43 unable to integrate problems

E - 0 integration timeouts